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UPCONING OF A SALT-WATER/FRESH-WATER INTERFACE BELOW A PUMPING WELL

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16. ABSTRACT

Analytical solutions for the upconing of an abrupt salt-water/fresh-water interface beneath a pumping well and for the concentration profile across a moving interface are developed for two types of upconing problems. The first considers the position of the interface and the salinity of the pumped water for a specified pumping rate. The second type of problem addresses the pumping schedules to prevent salinization of a well or to reach a predetermined salinity in the pumped water.

An interactive Fortran computer code has been developed to obtain solutions to both types of problems. The user is provided with options to modify the definition of a given problem, and, therefore, can gain some insight into the effects of geometry and physical properties on the rate and extent of upconing and the salinization of a well.

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FOREWORD

EPA is charged by Congress to protect the Nation's land, air, and water systems. Under a mandate of national environmental laws focused on air and water quality, solid waste management and the control of toxic substances, pesticides, noise, and radiation, the Agency strives to formulate and implement actions which lead to a compatible balance between human activities and the ability of natural systems to support and nurture life.

The Robert S. Kerr Environmental Research Laboratory is the Agency's center of expertise for investigation of the soil and subsurface environment. Personnel at the Laboratory are responsible for management of research programs to: (a) determine the fate, transport and transformation rates of pollutants in the soil, the unsaturated zone and the saturated zones of the subsurface environment; (b) define the processes to be used in characterizing the soil and subsurface environment as a receptor of pollutants; (c) develop techniques for predicting the effect of pollutants on ground water, soil and indigenous organisms; and (d) define and demonstrate the applicability and limitations of using natural processes, indigenous to the soil and subsurface environment, for the protection of this resource.

This project was initiated to develop an interactive computer model which could be utilized to predict the impact of density gradients on water flow. The model should be useful in regard to salt water intrusion and salt water injection problems. This model assumes idealized conditions of a homogeneous isotropic media. Care should be utilized if there is significant heterogeneity in the aquifer.

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ABSTRACT

Analytical solutions for the upconing of an abrupt salt-water/fresh-water interface beneath a pumping well and for the concentration profile across a moving interface are developed for two types of upconing problems. The first considers the position of the interface and the salinity of the pumped water for a specified pumping rate. The second type of problem addresses the pumping schedules to prevent salinization of a well or to reach a predetermined salinity in the pumped water.

An interactive Fortran computer code has been developed to obtain solutions to both types of problems. The user is provided with options to modify the definition of a given problem, and, therefore, can gain some insight into the effects of geometry and physical properties on the rate and extent of upconing and the salinization of a well.

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INTRODUCTION

Relatively simple analytical models can often be used to solve groundwater contamination problems, depending upon the complexity of the system and the availability of field data. Analytical models can also be used to gain some insight to the expected behavior of a complex system before progressing to the application of more sophisticated numerical models. In general, relatively few input parameters are required to define a problem using an analytical model and numerical results can be calculated in a few seconds. Analytical models are well suited for interactive use, and in some instances can be programmed on hand-held calculators.

This report presents an analytical solution for the upconing of an abrupt salt-water/fresh-water interface below a pumping well. Dispersion phenomena arising from the displacement of a moving interface, or a finite transition zone between the invading and displaced fluids, can be superimposed on the analytical solution for the position of an abrupt interface. An interactive Fortran computer code has been developed which enables the user to modify input parameters and to control the computational sequence. This interactive approach enables the user to gain insight into the effects of geometry and physical properties on the rate and extent of upconing and salinization of a well.

SECTION I

Mathematical Development

McWhorter (1972) presented the equations which describe the flow in saturated aquifers which are underlain by a zone of saline water and pointed out the difficulties in obtaining solutions to these problems. The complexity of the flow phenomenon has led many investigators to idealize the system as a fresh-water zone separated from an underlying salt-water zone by a sharp interface. In other words, the two fluids are assumed to be immiscible. Schmorak and Mercado (1969) followed this approach and accounted for the mixing of the two fluids by superimposing the effects of dispersion on the transient solution for the position of an abrupt interface.

Upconing of an Abrupt Interface

The following discussion is based on the studies of Bear and Dagan as reported by Schmorak and Mercado (1969). The basic assumptions underlying the theoretical development are: (1) the porous medium is homogeneous and nondeformable, (2) the two fluids are incompressible, immiscible, and separated by an abrupt interface (a geometric surface), and (3) the flow obeys Darcy's law. The non-linear boundary condition along the interface between the two fluids constitutes the major difficulty with the immiscible formulation of the problem. Bear and Dagan used the method of small pertubations to obtain an approximate solution for the position of the interface which served as a tool for obtaining analytical solutions for cases involving small deviations from an initially steady interface.

For the case of upconing beneath a pumping well partially penetrating a relatively thick confined aquifer as shown in Figure 1, Schmorak and

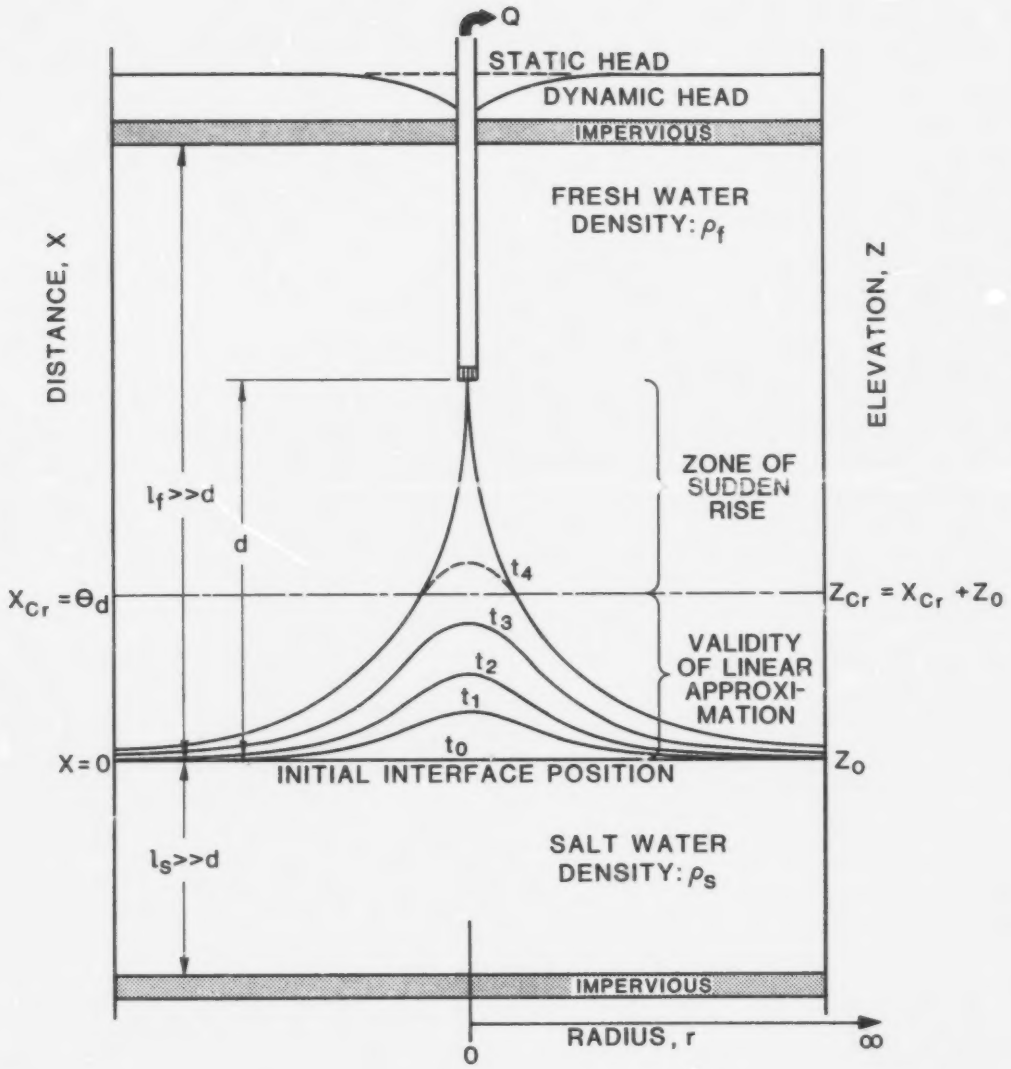

Figure 1. Upconing of an abrupt interface below a pumping well.

Mercado (1969) presented Bear and Dagan's solution for the position of the interface as a function of time and radial distance from the pumping well as

$$X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d}\left[\frac{1}{(1+R^2)^{\frac{1}{2}}} - \frac{1}{\left[(1+\tau)^2+R^2\right]^{\frac{1}{2}}}\right] \tag{1}$$

where R and τ are dimensionless distance and time parameters defined by

$$R = \frac{r}{d}\left(\frac{K_z}{K_x}\right)^{\frac{1}{2}} \tag{2}$$

and

$$\tau = \frac{(\Delta\rho/\rho)K_z}{2\theta d}\,t \tag{3}$$

Other notations are defined as follows (also refer to Figure 1):

d	distance from the bottom of the well to the initial interface elevation (L)
K_x, K_z	horizontal and vertical permeabilities, respectively (L/t)
Q	well pumping rate (L^3/t)
r	radial distance from well axis (L)
t	time elapsed since start of pumping (t)
X	rise of the interface above its initial position (L)
$\Delta\rho/\rho$	dimensionless density difference between the two fluids, $(\rho_s - \rho_f)/\rho_f$
θ	porosity of the aquifer.

Application of the method of small pertubations restricts changes in the interface elevation to relatively small values. In terms of the physical problem, this restriction implies $d \ll l_f$ and $d \ll l_s$. Although the governing differential equations have been formulated for a confined aquifer, the results can be applied to unconfined systems if the drawdown is negligible compared to the saturated thickness of the fresh-water zone.

The linear relationship, Equation 1, between the rise of the interface and the pumping rate is limited to a certain "critical rise," X_{cr}. This limitation arises from linear approximation of the boundary conditions. As the interface approaches this critical rise, the rate of rise increases. Above the critical rise the interface reaches the pumping well with a sudden jump. Muskat (1946) defines the zone of accelerated rise for $X/d > 0.48$ and the critical rise within the limits of $X/d \sim 0.60$ to 0.75. Schmorak and Mercado (1966) recommend application of the linear approximation for $X/d < \sim 0.5$. Sahni (1972) investigated the zone of instability of the interface using both numerical and physical models and recommended design criteria for skimming wells.

An abrupt interface such that (1) salinization of the pumping well occurs only for $X > X_{cr}=fd$ where f is the fractional critical rise, and (2) Equation 1 is valid for $0 \leq X \leq X_{cr}$ will be assumed in this report. Thus, the maximum permissible pumping rate which will ensure salt-free water can be obtained from Equation 1.

For $r = 0$ and $t \to \infty$

$$X(0,\infty) = \frac{Q}{2\pi d(\Delta\rho/\rho)K_x} \tag{4}$$

and

$$Q_{max} = 2\theta d(\Delta\rho/\rho)K_x X_{cr} \tag{5}$$

For a decaying mound, an imaginary recharge well is superimposed at time $t = t^*$ corresponding to the end of the pumping period, and for $t > t^*$

$$X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d}\left[\frac{1}{\left((1+\tau_1)^2 + R^2\right)^{\frac{1}{2}}} - \frac{1}{\left((1+\tau)^2 + R^2\right)^{\frac{1}{2}}}\right] \tag{6}$$

where

$$\tau_1 = \frac{(\Delta\rho/\rho)K_z}{2\theta d}\,(t-t^*) \qquad (7)$$

Values of R and τ are evaluated using Equations 2 and 3 respectively.

Dispersion

The upconing process as treated above assumes that the two fluids are immiscible and that the interface between them is abrupt. Actually, the interface is diffuse and a transition zone exists between the two fluids in which the concentration varies from the concentration in one fluid to the concentration in the other fluid over a finite distance. This transition zone is related to dispersion processes which alter the concentration profile across the moving interface.

Bear and Todd (1960) approximated the concentration profile as a function of position, X; the "interface" position, $\bar{X}$; the equivalent total distance the interface is displaced, $|\Delta\bar{X}|$, independent of direction; and the dispersivity, D_m. The correlation is given by

$$\varepsilon(X) = \frac{1}{2}\,\mathrm{ERFC}\left[\frac{X-\bar{X}}{2(D_m|\Delta\bar{X}|)^{\frac{1}{2}}}\right] \qquad (8)$$

where $\varepsilon(X)$ is a dimensionless, or relative, concentration defined as

$$\varepsilon(X) = \frac{C - C_b}{C_x - C_b} \qquad (9)$$

C = measured concentration at X

C_s = concentration of the invading fluid

C_b = background concentration of the displaced fluid.

Now, Equation 9 is a normal distribution function with a mean $\bar{X}$ and a standard deviation

$$\sigma = (2D_m|\Delta\bar{X}|)^{\frac{1}{2}} \qquad (10)$$

or

$$P_r \{X \geq x\} = \frac{1}{\sigma(2\pi)^{\frac{1}{2}}} \int_x^{\infty} \text{EXP} \left(- \frac{(X-\overline{X})^2}{2\sigma^2} \right) dx \tag{11}$$

From the definition of the error function,

$$P_r \{X \geq x\} = \frac{1}{2} \text{ERFC} \left(\frac{X-\overline{X}}{\sigma \sqrt{2}} \right) \tag{12}$$

The two-parameter distribution is completely defined once the mean, $\overline{X}$, and standard deviation, σ, are known.

The mean of the distribution is assumed to be the rise of the interface as determined from Equation 1, or

$$\overline{X} = X|_{\varepsilon=0.5} = X(r,t) \tag{13}$$

The standard deviation is defined as

$$2\sigma = \left(X|_{\varepsilon=0.841} - X|_{\varepsilon=0.159} \right) \tag{14}$$

Now, the width of the transition zone is a function of the total distance traveled, $|\overline{\Delta X}|$, (independent of direction) and the dispersivity as given by Equation 10, or

$$2\sigma = 2(2D_m|\overline{\Delta X}|)^{\frac{1}{2}} \tag{15}$$

For an interface with an initial transition width, $2\sigma_o$, when raised by a distance, $\overline{\Delta X}$,

$$\sigma_1 = \left(\sigma_o^2 + (2D_m|\overline{\Delta X}|) \right)^{\frac{1}{2}} \tag{16}$$

The concentration distribution function then becomes

$$\varepsilon(X) = \frac{1}{2} \text{ERFC} \left(\frac{X-\overline{X}}{\sigma_1 \sqrt{2}} \right) \tag{17}$$

or

$$\varepsilon(X) = \frac{1}{2} \text{ERFC} \left(\frac{X-\overline{X}}{\left(2\sigma_o^2 + 4D_m|\overline{\Delta X}| \right)^{\frac{1}{2}}} \right) \tag{18}$$

Two important points should be noted concerning the preceding discussion of dispersion. First, the "initial width" of the transition zone has been defined as two standard deviations of the dimensionless concentration distribution. This definition has been adopted for convenience and serves to define the standard deviation of the concentration distribution across the initial transition zone. Secondly, the dispersion concept should be limited to the zone below the critical depth. This point will be considered in more detail in the following paragraphs.

Superposition of Dispersion on the Upconing of an Abrupt Interface

The position of the interface as a function of time and radial distance from the well is evaluated using Equation 1, which assumes an abrupt interface between the two fluids. This elevation is assumed to correspond to $X|_{\varepsilon=0.5}$, or the mean of the concentration distribution across the transition zone. In other words,

$$\overline{X} = X(r,t) = \frac{Q}{2\pi(\Delta\rho/\rho)K_x d}\left(\frac{1}{\left((1+\tau_1)^2 + R^2\right)^{\frac{1}{2}}} - \frac{1}{\left((1+\tau)^2 + R^2\right)^{\frac{1}{2}}}\right) \qquad (6)$$

assuming an abrupt interface. The effect of dispersion arising from the displacement of the interface by a distance

$$|\overline{\Delta X}| = |X(r,t \leq t^*)| + |X(r,t > t^*)| \qquad (20)$$

is superimposed to estimate the concentration distribution across the interface using Equation 18, or

$$\varepsilon(X) = \frac{1}{2}\,\mathrm{ERFC}\left(\frac{X - \overline{X}}{\left(2\sigma_o^2 + 4D_m|\overline{\Delta X}|\right)^{\frac{1}{2}}}\right) \qquad (18)$$

The only difficulties in the approach occur for $\varepsilon(X) = 0.0$ and $\varepsilon(X) = 1.0$. Since

$$\varepsilon(X) = 0 \text{ for } X \to \infty$$

and

$$\varepsilon(X) = 1.0 \text{ for } X \to 0 \tag{22}$$

the transition zone would have an infinite width in theory. To overcome this physical impossibility, the width of the transition zone is arbitarily set at five standard deviations. This range includes approximately 99 percent of the area under the concentration distribution curve. Thus

$$\varepsilon(X) \simeq 0 \text{ for } X = \overline{X} + 2.5\ \sigma_1 \tag{23}$$

and

$$\varepsilon(X) \simeq 1 \text{ for } X = \overline{X} - 2.5\ \sigma_1 \tag{24}$$

Note that these limits differ from those used to define the "initial width" of the transition zone defined by Equation 14, or

$$2\sigma_o = \left(X\big|_{\varepsilon=0.841} - X\big|_{\varepsilon=0.159}\right) \tag{25}$$

Concentration in Pumped Water

The increase in concentration, or salinization, of pumped water is probably due to the intrusion of invading fluid above the critical depth. Data for two pumping tests on a coastal aquifer in the Ashqelon area of Israel indicated that the increase in salinity of the pumped water was approximately proportional to the average salinity above the critical depth (Schmorak and Mercado, 1969).

Previous discussion has emphasized that the linear approximation for the interface elevation is limited to elevations below the critical elevation and that the dispersion concept should be limited to the zone below the critical depth. The complex mixing and flow phenomena above the critical depth, near the well screen, and within the well pipe are approximated expirically using the approach followed by Schmorak and Mercado (1969).

The average dimensionless concentration of the transition zone above the critical rise, $\bar{\varepsilon}(X > X_{cr})$, is approximated as one-half the concentration at the critical depth, or

$$\varepsilon(X > X_{cr}) = 0.5\ \varepsilon(X_{cr}) \tag{26}$$

The concentration in the pumped water, ε_w, is determined from dilution of the average transition-zone concentration above the critical depth with displaced fluid, or

$$\varepsilon_w = \phi\ \bar{\varepsilon}(X > X_{cr}) \tag{27}$$

where ϕ is an interception coefficient, or the fraction of transition zone fluid in the total volume pumped.

SECTION II

Applications

Two types of upconing problems are considered. The first involves the description of the expected interface elevation and the salinity of the pumped water as a function of time for a given pumping rate. The second problem addresses the maximum rate at which a well can be pumped without exceeding a specified salinity in the pumped water. Both types of problems are discussed in the following paragraphs.

Case I - Estimation of Interface Elevations for a Given Pumping Rate

Case I problems are solved in a fairly straight-forward manner. Once the physical properties of the aquifer and the initial conditions have been specified, Equation 1 or Equation 6 is solved for the position of the abrupt interface, or the mean of the transition zone, i.e.,

$$X(r,t) = X\big|_{\varepsilon=0.5} = \overline{X} \tag{13}$$

In terms of elevations,

$$Z(r,t) = X(r,t) + Z_o \tag{28}$$

where Z_o is the initial interface elevation.

The concentration profile across the transition zone is evaluated using Equation 18. The salinity of the pumped water is determined from the dilution of the average transition zone salinity above the critical rise. From Equations 26 and 27

$$\varepsilon_w = 0.5 \; \phi \; \varepsilon(X_{cr}) \tag{29}$$

and from the definition of dimensionless concentration (Equation 9)

$$C_w = \varepsilon_w \; (C_s - C_b) + C_b \tag{30}$$

where C_w is the concentration in the pumped water.

Case II - Estimation the Maximum Permissible Pumping Rate to Prevent Salinization of a Well

Case II problems present some difficulty as the pumping rate, Q, is unknown. Thus, the elevation of the interface, $\overline{X}$, and the total displacement of the interface, $|\overline{\Delta X}|$, are also unknown. Equation 18 must be solved for the maximum permissible rise in interface elevation such that

$$\varepsilon(X_{cr}=fd) \leq \varepsilon_{max} \tag{31}$$

where

$$\varepsilon_{max} = \frac{\varepsilon_w}{0.5\phi} \tag{32}$$

Assuming a constant, steady pumping rate the total displacement of the interface will be equal to the rise of the interface or

$$|\overline{\Delta X}| = \overline{X}$$

and Equation 18 can be written as

$$\varepsilon_{max} = \frac{1}{2}\,\mathrm{ERFC}\left[\frac{X_{cr} - \overline{X}}{\left(2\sigma_o^2 + 4D_m\overline{X}\right)^{1/2}}\right] \tag{33}$$

Equation 33 must be solved for $\overline{X}$ using trial-and-error procedures. The maximum permissible rise in the interface elevation,

$$X_{max} \leq X_{cr}=fd \tag{34}$$

is then corrected for the concentration profile as

$$X^*_{max} = X_{max} - (fd-\overline{X}) \tag{35}$$

and

$$Z^*_{max} = X^*_{max} + Z_o \tag{36}$$

The maximum permissible steady-state pumping rate is then obtained using Equation 5, or

$$Q^*_{max} = 2\pi d(\Delta\rho/\rho)K_x X^*_{max} \tag{37}$$

where X^*_{max} depends only upon the critical rise and the dispersion pattern.

The time required to reach a predetermined salinity in the well can be estimated by rewriting Equation 1 as

$$t(C_w) = \frac{2\theta d}{(\Delta\rho/\rho)K_z}\left(\frac{1}{1 - \left(2\pi(\Delta\rho/\rho)K_x dX^*_{max}\right)/Q} - 1\right) \tag{38}$$

Substituting Equation 37 into Equation 38 yields

$$t(C_w) = \frac{2\theta d}{(\Delta\rho/\rho)K_z}\left(\frac{1}{1-Q^*_{max}/Q} - 1\right) \tag{39}$$

which can be used to estimate the time required to reach a predetermined salinity in the pumped water for pumping rates, Q, greater than the maximum steady-state pumping rate, Q^*_{max}.

An interactive computational code has been developed to calculate interface elevations, and concentrations for both Case I and Case II problems using the approach described above. The computer program is discussed in Section IV of this report.

Example Problem - Upconing Below a Coastal Collector Well

The application of the analytical model will be demonstrated using the field data for Test B on the coastal collector well Semadar 1 in the Ashqelon area of Israel (Schmorak and Mercado, 1969). Test B consisted of pumping Semadar 1 at a rate of 348 m^3/day for a period of 84 days. The upconing and decay of the salt-water/fresh-water interface were monitored by measuring the

TABLE 1

Parameters for Semadar 1 - Test B

Fresh-water density, ρ_f	1.00 g/cm^3
Salt-water density, ρ_s	1.03 g/cm^3
Porosity, θ	0.33
Horizontal permeability, K_x	14.7 m/day
Vertical permeability, K_z	14.7 m/day
Initial interface elevation, Z_o	-30.75 m MSL
Distance from bottom of well to initial interface, d	15.5 m
Fractional critical rise, f	0.4
Chloride concentration in salt water, C_s	22,000. ppm Cl
Chloride concentration in fresh water, C_b	145. ppm Cl
Dispersivity, D_m	0.5 m
Initial width of transition zone, $2\sigma_o$	3.5 m
Interception coefficient, ϕ	0.08
Pumping rate, Q	348.m^3/dy
Pumping period, t^*	84. dy

Observation wells:

Identification	Radius, m
T-2	4.5
T-3	12.4
T-4	16.7
T-5	33.9

salinity profiles in four observation wells. Samples of the pumped water were collected periodically and analyzed for chlorides. The properties of the fluids and the aquifer as estimated by Schmorak and Mercado are summarized in Table 1.

Program UPCONE was used to calculate the transient interface elevations and chloride concentrations in the pumped water. The input data dialog and printed results for this problem are presented in Appendix D.

The predicted and observed interface elevations after 16, 57, and 84 days of pumping are shown in Figure 2. The observed interface elevations correspond to elevations for a 50 percent relative concentration of sea water, or

$$\varepsilon = \frac{C - C_b}{C_s - C_b} = 0.5$$

The predicted and observed interface elevations match fairly well with the exception of the values for observation well T-2. Well T-2 is located 4.5 meters from the pumping well and penetrates to an elevation of 31.10 meters below MSL. However, this well is screened to an elevation of only 29.02 meters below MSL, and Schmorak and Mercado (1969) indicate that saline water was entrapped at the bottom of the well from a previous pumping test. The predicted rise of the interface at well T-2 also approaches the critical elevation for a fractional critical rise of 0.4. Thus, the observed interface elevation could be in a zone of accelerated rise. Finally, the reported concentration gradients were very steep in well T-2, and small errors in concentration measurements could lead to large errors in estimating the position of the interface.

The predicted upconing and recovery curves for each of the four observation wells are shown in Figure 3. With the exception of well T-2 the

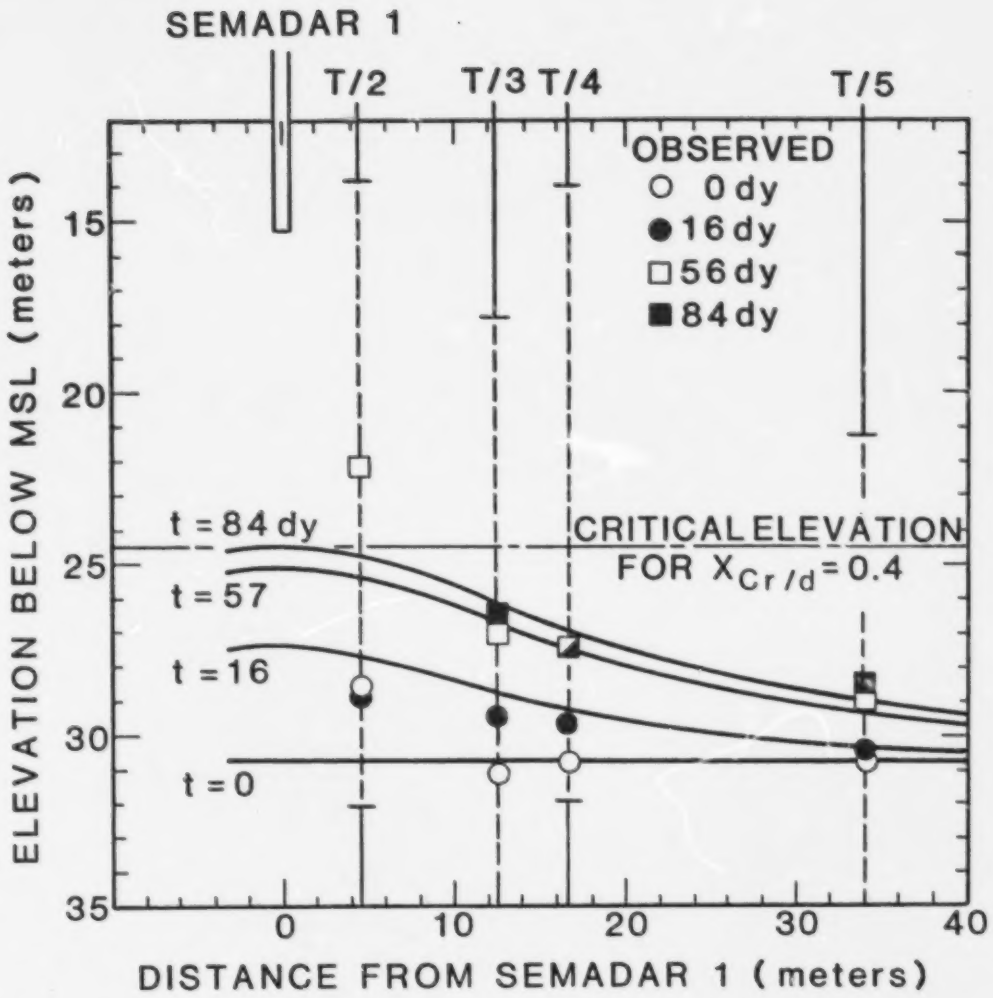

Figure 2. Predicted and observed interface elevations for Test B of Semadar 1.

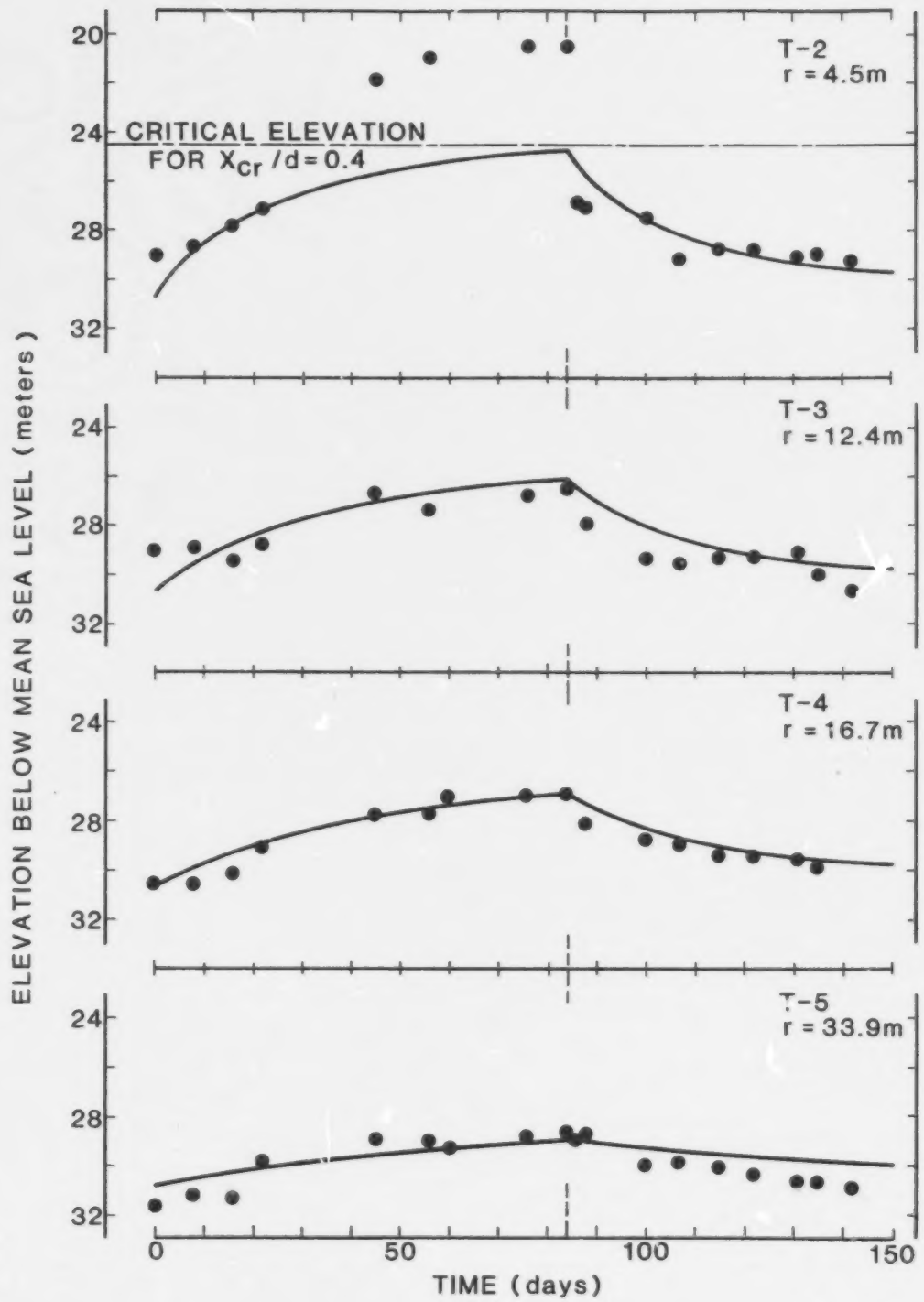

Figure 3. Predicted and observed upconing and recovery curves for Test B of Semadar 1.

predicted upconing curves follow the observed interface elevations fairly well. The recession curves for all four observation wells are also in fair agreement with the field data.

Initial observed relative concentrations for wells T-3, T-4, and T-5 are plotted on Figures 4a and 4b. The predicted concentration profile across the transition zone using an initial transition zone width, $2\sigma_o$, of 3.5 meters is also shown. This value represents an average of the widths of the transition zones at the three observation wells.

The parameters listed in Table 1 were used to predict the concentration of chlorides in the pumped water as a function of time. The results of the simulation are summarized in Figure 5 and agree very well with the observed values.

No effort has been made in this report to "optimize" model input parameters or to quantify the "goodness of fit" between observed and predicted values of elevations, concentration profiles, or salinity of the pumped water. However, a qualitive comparison of the predicted and observed values as shown in Figures 2, 3, and 5 indicate that the assumptions incorporated in the analytical model approach the field conditions.

Program UPCONE was also used to develop salinity/maximum pumping rate relationships and salinity/time relationships for Semadar 1. These relationships correspond to Case II types of problems. The corrected critical interface elevations, Z^*_{max}, and maximum steady-state pumping rates, Q^*_{max}, for several values of predetermined salinity in the pumped water are presented in Table 2. The time required to reach the specified salinity in the pumped water were also calculated for two optional pumping rates. These pumping rates correspond to Test A and Test B of Semadar 1.

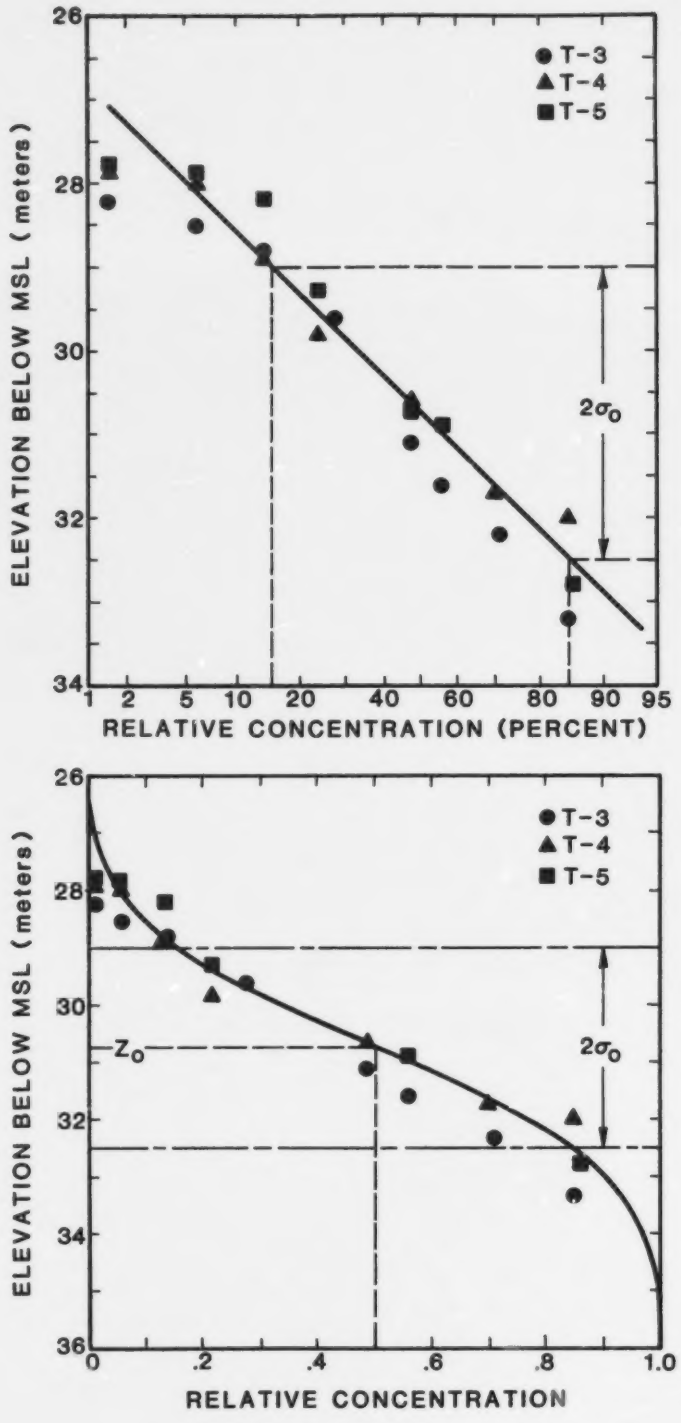

Figure 4. Concentration profile across initial transition zone.
(a) Relative concentration on probability scale.
(b) Relative concentration on arithmetic scale.

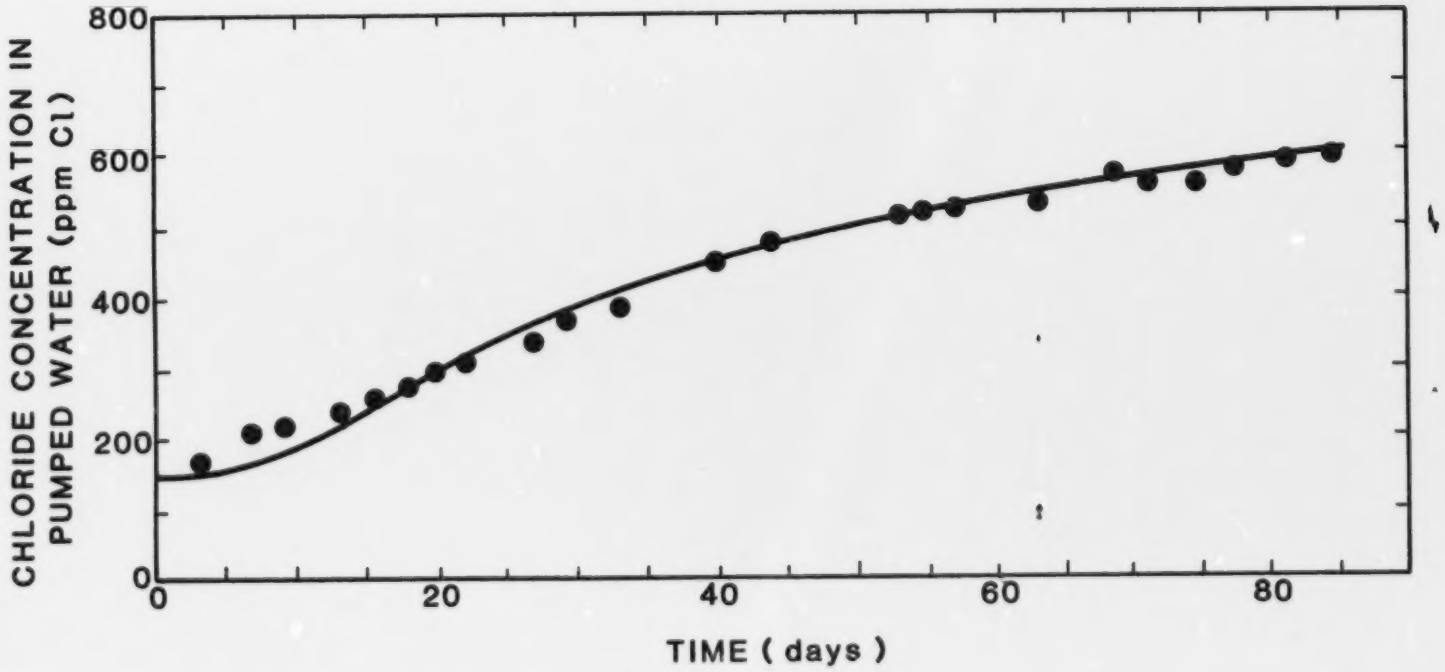

Figure 5. Predicted and observed chloride concentrations in pumped water for Test B on Semadar 1.

TABLE 2

Salinity/Maximum Pumping Rate and Salinity/Time

Relationships For Semadar 1 ($X_{cr} = 0.4d$)

Relative Concentration of Salt Water (Dimensionless)	Chloride Concentration In Pumped Water (ppm CL)	X^*_{max} (meters)	Q^*_{max} (m^3/day)	Time	
				Test A $Q = 575\ m^3/day$ (days)	Test B $Q = 348\ m^3/day$ (days)
0.001	166.85	2.85	79.57	3.73	6.88
.003	210.56	3.73	117.40	5.95	11.81
.005	254.27	4.30	141.66	7.58	15.92
.010	363.55	5.36	187.34	11.21	27.05
.020	582.10	7.20	266.28	20.01	75.59
.030	800.65	9.49†	364.76†	40.25	-

(†) $X^*_{max} > X_{cr} = 6.20$ meters

The example problems using the data for Test B of Semadar 1 described above are intended to support, in general, the validity of the theoretical approach and to demonstrate the application of the analytical model to a typical upconing problem. The reader interested in methods which might be used to develop input parameters for the model are referred to the Schmorak and Mercado (1969) discussion of the field investigation and interpretation of the field data.

SECTION III

Program UPCONE

Program UPCONE evaluates the position of an abrupt salt-water/fresh-water interface beneath a pumping well as a function of time and radial distance from the well. The program has also been written to (1) superimpose the effects of dispersion on the abrupt interface to estimate the concentration profile across the interface and the salinity of the pumped water, or (2) estimate the maximum pumping rate or time required to reach a specified salinity in the well. The program has been designed for interactive use and requires data input under two modes of operation--"Basic Input Data" and "Edit."

Basic Input Data

Basic input data are required to initiate a new problem using the UPCONE program. The data entries include the problem title, the physical properties of the aquifer and the two fluids, and the geometry of the system. The user is prompted for the required data through a series of input commands described below. Numeric data should be entered through the keyboard with decimal points, and multiple data entries should be separated by a comma(s). The first basic input command is

```
ENTER TITLE
?
```

Any valid keyboard characters can be used. The first 60 characters will be retained for further problem identification.

The next two input commands are used to define the units of all variables used in the calculations and printouts of the results. Any consistent set of units may be used. The two commands are

```
ENTER UNITS FOR LENGTH (2 CHARACTERS)
?
ENTER UNITS FOR TIME (2 CHARACTERS)
?
```

Any valid keyboard characters can be used. The first two characters will be retained for identification of length and time units.

The next series of input commands are used to specify the physical properties of the fluids and the aquifer. Input data errors which may interrupt the computational sequence are detected by the program and a command is issued to reenter the data for the appropriate variable. The series of commands are as follows:

```
ENTER FRESH-WATER AND SALT-WATER DENSITIES
?,?
```

The densities, or specific gravities, of the two fluids may be entered in any units so long as the units are identical for both entries.

```
ENTER AQUIFER POROSITY
?
```

Enter the volume void fraction as a decimal value greater than zero and less than one.

```
ENTER HORIZONTAL PERMEABILITY (L/t)
?
```

ENTER VERTICAL PERMEABILITY (L/t)

?

Horizontal and vertical permeabilities must be entered with dimensions of L/t in the units requested. Numerical values for both entries must be greater than zero.

ENTER INITIAL INTERFACE ELEVATION (L)

?

The initial interface elevation may be either positive, zero, or negative, depending upon the location of the reference elevation with respect to the initial abrupt interface (or the elevation of the mean concentration of the transition zone). The elevation must be entered in the units requested.

ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE (L)

?

The entry must be positive and in the units requested.

ENTER FRACTIONAL CRITICAL RISE

?

The fractional critical rise must be a decimal value greater than zero and less than one.

The next basic input command is used to select the option of performing concentration calculations. The command is

CONCENTRATION CALCULATIONS? (Y/N)

If the user does not respond with Y, the problem title and parameters which have been specified are listed. The critical rise and the maximum steady-state

pumping rate which will maintain the interface at the critical elevation are evaluated and the results are printed. The program then enters the "Edit" mode.

If the response to the last basic input command is Y, the user will be requested for additional basic input data required to carry out the concentration calculations. The first command is

ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)

?

Any valid keyboard characters can be used. The first 6 characters will be used to specify concentrations for following data entries and printed results. The following commands are:

ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (M/L^3)

?,?

The concentration of any desired component in the invading fluid and the displaced fluid are entered. If the user wishes to work in terms of dimensionless concentrations, enter a value of 1.0 for the salt-water concentration and a value of 0.0 for the background concentration.

ENTER DISPERSIVITY (L)

?

The dispersivity has dimensions of L and must be entered in the units requested. Numerical values must be greater than zero.

ENTER INITIAL WIDTH OF TRANSITION ZONE (M)

?

The width of the transition zone is defined as two standard deviations of the concentration distribution function across the transition zone, as discussed in Section II of this report. For an initially abrupt interface enter a value of zero.

ENTER INTERCEPTION COEFFICIENT

?

The interception coefficient is the fraction of transition zone water in the total volume pumped. Enter a decimal fraction greater than zero and less than one.

At this point the program will list the problem title and parameters as they are currently specified along with the critical rise and the maximum permissible pumping rate for an abrupt interface. The program then enters the "Edit" mode.

Edit Commands

Once the basic input data have been entered, the problem as currently defined is listed and the program enters the "Edit" mode. The edit commands are listed in Table 3. The request for information is

ENTER NEXT COMMAND

?

One of the responses from Table 1 should be given. If the response is incorrect or improperly formated the statement

ERROR IN LAST COMMAND--REENTER

?

is issued. Error messages for invalid numeric data will be issued as described under Basic Input Data.

The request for information will be repeated until one of the responses EL, PR, LI, NP or DN is entered.

EL will initiate the calculation of interface elevations for a specified pumping rate. The user is given the option of calculating the concentration profile across the transition zone by responding to the following command

CONCENTRATION CALCULATIONS ? (Y/N)

If the response is Y and the data required for concentration calculations have not been entered previously, the user is prompted for the required information using "Basic Input" commands.
For the initial use of the EL edit command, the following requests for data are issued:

ENTER PUMPING RATE (CU L/t) AND PERIOD (t)
?,?

Enter the well pumping rate and the pumping period in the units requested. Both entries must be positive and separated by a comma.
Two additional requests for data are used to define the coordinates of the observation points. The first is

ENTER TFIRST, TLAST, DELTAT (t)
?,?,?

Input units for the time variables must be in the units requested. TFIRST must not be negative value. A zero entry for DELTAT will result in interface elevations at a single time. The second request is

ENTER RFIRST, RLAST, DELTAR (L)

?,?,?

The numerical values used to define the radial coordinates of the observation points may be positive or negative. The results of the calculations will be printed from RFIRST to RLAST.

The pumping rate and observation coordinate parameters are listed and a request

CONTINUE ? (Y/N)

is issued. If the response is not Y, the program returns to the edit mode. If the response is Y, the program proceeds to the computation of interface elevations at the specified times and radial distances from the well and prints the results. If concentration calculations were requested, the concentration in the pumped water and the concentration distribution across the transition zone are also evaluated and printed for the specified times.

On subsequent use of the EL edit command, the pumping rate and observation coordinate data as currently defined will be listed and a request to continue will be issued.

PR initiates the calculation of maximum permissible interface elevations and pumping rates for a specified concentration in the pumped water. If the data required for concentration have not been entered previously, the user is prompted for the required data using "Basic Input Data" dialog. The following request for information is then issued:

ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (M/L^3)

?

Enter the concentration in the units requested. The numerical value must not be negative.

The program then lists the problem as currently defined and evaluates the maximum permissible interface elevation and pumping rate. The following request for information is then issued:

ENTER OPTIONAL PUMPING RATE (L^3/t)

?

If a value greater than the maximum pumping rate is entered, the time to reach the specified concentration in the pumped water will be calculated, and the command reissued. If a value less than the maximum pumping rate is entered the program returns to the edit mode.

LI will list the problem as currently defined.

NP will request a complete new problem definition using the "Basic Input Data" dialog.

DN will terminate the program.

Although many tests for valid input data and properly formulated edit commands have been embedded in the computer code, the user is encouraged to correct "keyboard errors" before the data are transmitted. This practice will serve to minimize the frustration of program termination as a result of fatal errors during execution of the numerical computations.

TABLE 3

Edit Commands for UPCONE

Command	Variable Changed/Execution
CO	Salt-water and background concentrations
CR	Fractional critical rise
DI	Dispersivity
DT	Depth from bottom of well to initial interface
FD	Fluid densities
IC	Interception coefficient
KX	Horizontal permeability
KZ	Vertical permeability
OB	Time and radius coordinates
PO	Porosity
QP	Pumping rate and time
RC	Radius coordinates
TC	Time coordinates
TW	Initial width of transition zone
ZO	Initial interface elevation
EL	Interface elevation calculations
DN	Terminate program
LI	List problem definition
NP	New problem
PR	Pumping rate calculations

REFERENCES

Abramowitz, M. and I. A. Stegun. 1966. Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. National Bureau of Standards Applied Mathematics Series 55, U. S. Department of Commerce, 1046 pp.

Bear, J. and D. K. Todd. 1960. "The Transition Zone between Fresh and Salt Waters in Coastal Aquifers." Contribution No. 29, Water Resources Center, University of California, Berkeley, CA.

Carnahan, B., H. A. Luther and J. O. Wilkes. 1969. Applied Numerical Methods. John Wiley and Sons, New York, NY.

McWhorter, D. B. 1972. "Steady and Unsteady Flow of Fresh Water in Saline Aquifers." Water Management Technical Report No. 20, Engineering Research Center, Colorado State University, Fort Collins, CO.

Schmorak, S. and A. Mercado. 1969. "Upconing of Fresh Water-Sea Water Interface Below Pumping Wells, Field Study." Water Resources Research, Vol. 5, No. 6, pp 1290-1311.

APPENDIX A

Example Problems

The following pages contain the documentation of the upconing simulation used to generate the predicted interface/time and concentration/time relationships for the example problem discussed in Section II of this report.

```
 ENTER TITLE
 ?
SEMADAR 1 -- TEST B
 ENTER UNITS FOR LENGTH (2 CHARACTERS)
 ?
M
 ENTER UNITS FOR TIME (2 CHARACTERS)
 ?
DY
 ENTER FRESH-WATER AND SALT-WATER DENSITIES
 ?,?
1.00,1.03
 ENTER AQUIFER POROSITY
 ?
0.33
 ENTER HORIZONTAL PERMEABILITY (M /DY)
 ?
14.7
 ENTER VERTICAL PERMEABILITY (M /DY)
 ?
14.7
 ENTER INITIAL INTERFACE ELEVATION (M )
 ?
-30.75
 ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE (M )
 ?
15.5
 ENTER FRACTIONAL CRITICAL RISE
 ?
0.4


 CONCENTRATION CALCULATIONS ? (Y/N)
N
```

```
   SEMADAR 1 -- TEST B

     DENSITY OF FRESH WATER                              1.0000
     DENSITY OF SALT WATER                               1.0300

     AQUIFER POROSITY                                     .3300
     HORIZONTAL PERMEABILITY (M /DY)                    14.7000
     VERTICAL PERMEABILITY (M /DY)                      14.7000

     INITIAL INTERFACE ELEVATION (M )                  -30.7500
     DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )     15.5000
     FRACTIONAL CRITICAL RISE                             .4000
     CRITICAL RISE (M )                                 6.2000
     CRITICAL ELEVATION (M )                           -24.5500

     MAXIMUM STEADY-STATE PUMPING RATE (CU M /DY)      266.2818



  ENTER NEXT COMMAND
  ?
EL


  CONCENTRATION CALCULATIONS ? (Y/N)
N


  ENTER PUMPING RATE (CU M /DY) AND PERIOD (DY)
  ?,?
348.,84.
  ENTER TFIRST, TLAST, DELTAT (DY)
  ?,?,?
0.,57.,16.
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
0.,40.,5.


     PUMPING RATE (CU M /DY)                           348.0000
     PUMPING PERIOD (DY)                                84.0000

     TFIRST =      .0000   TLAST =   57.0000   DELTAT =   16.0000
     RFIRST =      .0000   RLAST =   40.0000   DELTAR =    5.0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN      75.59 DY

  CONTINUE ? (Y/N)
Y
```

PUMPING RATE: 348.00 CU M /DY FOR 84.00 DY

INTERFACE ELEVATIONS (M)

T (DY) \ R (M)	.00	5.00	10.00	15.00	20.00	25.00	30.00
.00	-30.75	-30.75	-30.75	-30.75	-30.75	-30.75	-30.75
16.00	-27.44	-27.75	-28.42	-29.09	-29.60	-29.95	-30.18
32.00	-26.05	-26.41	-27.23	-28.08	-28.78	-29.30	-29.67
48.00	-25.29	-25.66	-26.52	-27.45	-28.22	-28.82	-29.26
57.00	-24.99	-25.37	-26.25	-27.18	-27.98	-28.60	-29.08

INTERFACE ELEVATIONS (M) (CONTINUED)

T (DY)	R (M) 35.00	40.00
.00	-30.75	-30.75
16.00	-30.34	-30.45
32.00	-29.94	-30.13
48.00	-29.60	-29.84
57.00	-29.43	-29.70

```
  ENTER NEXT COMMAND
  ?
DB
  ENTER TFIRST, TLAST, DELTAT (DY)
  ?,?,?
0.,160.,5.
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
4.5,0.,0.


  ENTER NEXT COMMAND
  ?
EL


  CONCENTRATION CALCULATIONS ? (Y/N)
N

     PUMPING RATE (CU M /DY)                          348.0000
     PUMPING PERIOD (DY)                               84.0000

     TFIRST =      .0000   TLAST =  160.0000   DELTAT =    5.0000
     RFIRST =    4.5000    RLAST =     .0000   DELTAR =     .0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN     75.59 DY

  CONTINUE ? (Y/N)
Y
```

```
                PUMPING RATE:      348.00 CU M /DY FOR     84.00 DY

                INTERFACE ELEVATIONS (M )

   *
     * R (M )
       *          4.50
  T (DY) *
         *
        .00    -30.75
       5.00    -29.45
      10.00    -28.52
      15.00    -27.81
      20.00    -27.27
      25.00    -26.83
      30.00    -26.47
      35.00    -26.18
      40.00    -25.93
      45.00    -25.71
      50.00    -25.53
      55.00    -25.36
      60.00    -25.22
      65.00    -25.09
      70.00    -24.98
      75.00    -24.88
      80.00    -24.79
      85.00    -25.00
      90.00    -26.13
      95.00    -26.94
     100.00    -27.55
     105.00    -28.01
     110.00    -28.37
     115.00    -28.67
     120.00    -28.91
     125.00    -29.11
     130.00    -29.27
     135.00    -29.41
     140.00    -29.54
     145.00    -29.64
     150.00    -29.73
     155.00    -29.81
     160.00    -29.88

  * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


  ENTER NEXT COMMAND
  ?
RC
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
12.4,0.,0.


  ENTER NEXT COMMAND
  ?
EL


  CONCENTRATION CALCULATIONS ? (Y/N)
N

     PUMPING RATE (CU M /DY)                          348.0000
     PUMPING PERIOD (DY)                               84.0000

     TFIRST =      .0000   TLAST =  160.0000   DELTAT =    5.0000
     RFIRST =   12.4000    RLAST =     .0000   DELTAR =     .0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN     75.59 DY

  CONTINUE ? (Y/N)
Y
```

```
                PUMPING RATE:      348.00 CU M /DY FOR     84.00 DY

                INTERFACE ELEVATIONS (M )

   *
     * R (M )
     *          12.40
  T (DY) *
         *
         .00   -30.75
        5.00   -29.99
       10.00   -29.36
       15.00   -28.85
       20.00   -28.42
       25.00   -28.06
       30.00   -27.76
       35.00   -27.50
       40.00   -27.28
       45.00   -27.08
       50.00   -26.91
       55.00   -26.76
       60.00   -26.63
       65.00   -26.51
       70.00   -26.40
       75.00   -26.30
       80.00   -26.22
       85.00   -26.30
       90.00   -26.96
       95.00   -27.49
      100.00   -27.92
      105.00   -28.28
      110.00   -28.57
      115.00   -28.82
      120.00   -29.02
      125.00   -29.20
      130.00   -29.34
      135.00   -29.47
      140.00   -29.58
      145.00   -29.68
      150.00   -29.77
      155.00   -29.84
      160.00   -29.91

  * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


  ENTER NEXT COMMAND
  ?
RC
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
16.7,0.,0.


  ENTER NEXT COMMAND
  ?
EL


  CONCENTRATION CALCULATIONS ? (Y/N)
N


     PUMPING RATE (CU M /DY)                          348.0000
     PUMPING PERIOD (DY)                               84.0000

     TFIRST =      .0000   TLAST =  160.0000   DELTAT =    5.0000
     RFIRST =   16.7000    RLAST =     .0000   DELTAR =     .0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN     75.59 DY

  CONTINUE ? (Y/N)
Y
```

```
                PUMPING RATE:      348.00 CU M /DY FOR     84.00 DY

                INTERFACE ELEVATIONS (M )

   *
     * R (M )
       *          16.70
  T (DY) *
           *
         .00    -30.75
        5.00    -30.23
       10.00    -29.76
       15.00    -29.36
       20.00    -29.00
       25.00    -28.70
       30.00    -28.44
       35.00    -28.21
       40.00    -28.00
       45.00    -27.83
       50.00    -27.67
       55.00    -27.53
       60.00    -27.40
       65.00    -27.29
       70.00    -27.19
       75.00    -27.09
       80.00    -27.01
       85.00    -27.04
       90.00    -27.48
       95.00    -27.87
      100.00    -28.20
      105.00    -28.49
      110.00    -28.73
      115.00    -28.94
      120.00    -29.12
      125.00    -29.27
      130.00    -29.41
      135.00    -29.52
      140.00    -29.63
      145.00    -29.72
      150.00    -29.80
      155.00    -29.87
      160.00    -29.93

  * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


  ENTER NEXT COMMAND
  ?
RC
  ENTER RFIRST, RLAST, DELTAR (M )
  ?,?,?
33.9


  ENTER NEXT COMMAND
  ?
EL


  CONCENTRATION CALCULATIONS ? (Y/N)
N


     PUMPING RATE (CU M /DY)                          348.0000
     PUMPING PERIOD (DY)                               84.0000

     TFIRST =      .0000   TLAST =  160.0000   DELTAT =    5.0000
     RFIRST =   33.9000   RLAST =     .0000   DELTAR =     .0000

  NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN     75.59 DY

  CONTINUE ? (Y/N)
Y
```

```
                PUMPING RATE:      348.00 CU M /DY FOR     84.00 DY

                INTERFACE ELEVATIONS (M )

  *
    * R (M )
      *          33.90
T (DY) *
       *
       .00    -30.75
      5.00    -30.62
     10.00    -30.48
     15.00    -30.34
     20.00    -30.20
     25.00    -30.07
     30.00    -29.94
     35.00    -29.82
     40.00    -29.70
     45.00    -29.59
     50.00    -29.49
     55.00    -29.40
     60.00    -29.31
     65.00    -29.23
     70.00    -29.15
     75.00    -29.08
     80.00    -29.02
     85.00    -28.98
     90.00    -29.05
     95.00    -29.14
    100.00    -29.23
    105.00    -29.32
    110.00    -29.41
    115.00    -29.49
    120.00    -29.58
    125.00    -29.65
    130.00    -29.72
    135.00    -29.79
    140.00    -29.85
    145.00    -29.91
    150.00    -29.96
    155.00    -30.01
    160.00    -30.05

 * NOTE: CRITICAL ELEVATION OF  -24.55 M  EXCEEDED AT R=0 AND T=   75.59 DY


 ENTER NEXT COMMAND
 ?
OB
 ENTER TFIRST, TLAST, DELTAT (DY)
 ?,?,?
0.,84.,5.
 ENTER RFIRST, RLAST, DELTAR (M )
 ?,?,?
0.


 ENTER NEXT COMMAND
 ?
EL


 CONCENTRATION CALCULATIONS ? (Y/N)
Y
 ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)
 ?
PPM CL
 ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (PPM CL)
 ?,?
22000.,145.
 ENTER DISPERSIVITY (M )
 ?
0.5
 ENTER INITIAL WIDTH OF TRANSITION ZONE (M )
 ?
3.5
 ENTER INTERCEPTION COEFFICIENT
 ?
0.08
```

```
SEMADAR 1 -- TEST B

  DENSITY OF FRESH WATER                              1.0000
  DENSITY OF SALT WATER                               1.0300

  AQUIFER POROSITY                                     .3300
  HORIZONTAL PERMEABILITY (M /DY)                    14.7000
  VERTICAL PERMEABILITY (M /DY)                      14.7000

  INITIAL INTERFACE ELEVATION (M )                  -30.7500
  DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )     15.5000
  FRACTIONAL CRITICAL RISE                             .4000
  CRITICAL RISE (M )                                 6.2000
  CRITICAL ELEVATION (M )                           -24.5500

  MAXIMUM STEADY-STATE PUMPING RATE (CU M /DY)      266.2818



  CONCENTRATION IN SALT WATER (PPM CL)            22000.0000
  BACKGROUND CONCENTRATION (PPM CL)                 145.0000
  INITIAL WIDTH OF TRANSITION ZONE (M )               3.5000

  DISPERSIVITY (M )                                    .5000
  INTERCEPTION COEFFICIENT                             .0800



  PUMPING RATE (CU M /DY)                           348.0000
  PUMPING PERIOD (DY)                                84.0000

  TFIRST =      .0000   TLAST =   84.0000   DELTAT =    5.0000
  RFIRST =      .0000   RLAST =     .0000   DELTAR =     .0000

NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION IN     75.59 DY

CONTINUE ? (Y/N)
Y
```

PUMPING RATE: 348.00 CU M /DY FOR 84.00 DY

INTERFACE ELEVATIONS (M)

T (DY)	R (M) .00
.00	-30.75
5.00	-29.31
10.00	-28.31
15.00	-27.57
20.00	-27.00
25.00	-26.55
30.00	-26.18
35.00	-25.88
40.00	-25.62
45.00	-25.40
50.00	-25.22
55.00	-25.05
60.00	-24.91
65.00	-24.78
70.00	-24.66
75.00	-24.56
80.00	-24.47*
84.00	-24.40*

* NOTE: CRITICAL ELEVATION OF -24.55 M EXCEEDED AT R=0 AND T= 75.59 DY

CONCENTRATION IN WELL AND PROFILES BENEATH WELL (PPM CL)

T (DY)	C(WELL) E(WELL)	ELEVATION FOR C/(E) (M) 145.0 (.0)	2330.5 (.1)	4516.0 (.2)	6701.5 (.3)	8887.0 (.4)	11072.5 (.5)
.00	145.17 (.0000)	-26.4	-28.5	-29.3	-29.8	-30.3	-30.8
5.00	155.81 (.0005)	-24.0*	-26.6	-27.5	-28.2	-28.8	-29.3
10.00	192.67 (.0022)	-22.4*	-25.3	-26.3	-27.1	-27.7	-28.3
15.00	244.28 (.0045)	-21.3*	-24.4*	-25.5	-26.3	-26.9	-27.6
20.00	297.22 (.0070)	-20.5*	-23.7*	-24.8	-25.6	-26.3	-27.0
25.00	345.51 (.0092)	-19.8*	-23.1*	-24.3*	-25.1	-25.9	-26.5
30.00	387.60 (.0111)	-19.3*	-22.6*	-23.9*	-24.7	-25.5	-26.2
35.00	423.69 (.0128)	-18.8*	-22.3*	-23.5*	-24.4*	-25.2	-25.9
40.00	454.52 (.0142)	-18.5*	-22.0*	-23.2*	-24.1*	-24.9	-25.6
45.00	480.92 (.0154)	-18.2*	-21.7*	-23.0*	-23.9*	-24.7	-25.4
50.00	503.65 (.0164)	-17.9*	-21.5*	-22.7*	-23.7*	-24.5*	-25.2
55.00	523.35 (.0173)	-17.7*	-21.3*	-22.6*	-23.5*	-24.3*	-25.1
60.00	540.53 (.0181)	-17.4*	-21.1*	-22.4*	-23.3*	-24.2*	-24.9
65.00	555.62 (.0188)	-17.3*	-20.9*	-22.2*	-23.2*	-24.0*	-24.8
70.00	568.94 (.0194)	-17.1*	-20.8*	-22.1*	-23.1*	-23.9*	-24.7
75.00	580.79 (.0199)	-17.0*	-20.7*	-22.0*	-23.0*	-23.8*	-24.6
80.00	591.38 (.0204)	-16.8*	-20.6*	-21.9*	-22.9*	-23.7*	-24.5*
84.00	599.06 (.0208)	-16.7*	-20.5*	-21.8*	-22.8*	-23.6*	-24.4*

* NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW THE CRITICAL ELEVATION OF -24.5500 M

CONCENTRATION IN WELL AND PROFILES BENEATH WELL (PPM CL)

T (DY)	C(WELL) E(WELL)	ELEVATION FOR C/(E) (M) 11072.5 (.5)	13258.0 (.6)	15443.5 (.7)	17629.0 (.8)	19814.5 (.9)	22000.0 (1.0)
.00	145.17 (.0000)	-30.8	-31.2	-31.7	-32.2	-33.0	-35.1
5.00	155.81 (.0005)	-29.3	-29.9	-30.4	-31.1	-32.0	-34.6
10.00	192.67 (.0022)	-28.3	-28.9	-29.5	-30.3	-31.3	-34.2
15.00	244.28 (.0045)	-27.6	-28.2	-28.9	-29.7	-30.8	-33.8
20.00	297.22 (.0070)	-27.0	-27.7	-28.4	-29.2	-30.3	-33.5
25.00	345.51 (.0092)	-26.5	-27.2	-28.0	-28.8	-30.0	-33.3
30.00	387.60 (.0111)	-26.2	-26.9	-27.6	-28.5	-29.7	-33.1
35.00	423.69 (.0128)	-25.9	-26.6	-27.4	-28.2	-29.5	-32.9
40.00	454.52 (.0142)	-25.6	-26.3	-27.1	-28.0	-29.3	-32.8
45.00	480.92 (.0154)	-25.4	-26.1	-26.9	-27.8	-29.1	-32.7
50.00	503.65 (.0164)	-25.2	-26.0	-26.8	-27.7	-29.0	-32.5
55.00	523.35 (.0173)	-25.1	-25.8	-26.6	-27.5	-28.8	-32.5
60.00	540.53 (.0181)	-24.9	-25.7	-26.5	-27.4	-28.7	-32.4
65.00	555.62 (.0188)	-24.8	-25.5	-26.4	-27.3	-28.6	-32.3
70.00	568.94 (.0194)	-24.7	-25.4	-26.3	-27.2	-28.5	-32.2
75.00	580.79 (.0199)	-24.6	-25.3	-26.2	-27.1	-28.5	-32.2
80.00	591.38 (.0204)	-24.5*	-25.2	-26.1	-27.0	-28.4	-32.1
84.00	599.06 (.0208)	-24.4*	-25.2	-26.0	-27.0	-28.3	-32.1

* NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW
THE CRITICAL ELEVATION OF -24.5500 M

ENTER NEXT COMMAND
?
NP

```
 ENTER TITLE
 ?
SEMADAR 1  SALINITY/TIME RELATIONSHIPS
 ENTER UNITS FOR LENGTH (2 CHARACTERS)
 ?
M
 ENTER UNITS FOR TIME (2 CHARACTERS)
 ?
DY
 ENTER FRESH-WATER AND SALT-WATER DENSITIES
 ?,?
1.00,1.03
 ENTER AQUIFER POROSITY
 ?
0.33
 ENTER HORIZONTAL PERMEABILITY (M /DY)
 ?
14.7
 ENTER VERTICAL PERMEABILITY (M /DY)
 ?
14.7
 ENTER INITIAL INTERFACE ELEVATION (M )
 ?
-30.75
 ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE (M )
 ?
15.5
 ENTER FRACTIONAL CRITICAL RISE
 ?
0.4


 CONCENTRATION CALCULATIONS ? (Y/N)
Y
 ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)
 ?
PPM CL
 ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (PPM CL)
 ?,?
22000.,145.
 ENTER DISPERSIVITY (M )
 ?
0.5
 ENTER INITIAL WIDTH OF TRANSITION ZONE (M )
 ?
3.5
 ENTER INTERCEPTION COEFFICIENT
 ?
0.08
```

```
SEMADAR 1  SALINITY/TIME RELATIONSHIPS

  DENSITY OF FRESH WATER                              1.0000
  DENSITY OF SALT WATER                               1.0300

  AQUIFER POROSITY                                     .3300
  HORIZONTAL PERMEABILITY (M /DY)                    14.7000
  VERTICAL PERMEABILITY (M /DY)                      14.7000

  INITIAL INTERFACE ELEVATION (M )                  -30.7500
  DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )     15.5000
  FRACTIONAL CRITICAL RISE                             .4000
  CRITICAL RISE (M )                                 6.2000
  CRITICAL ELEVATION (M )                          -24.5500

  MAXIMUM STEADY-STATE PUMPING RATE (CU M /DY)     266.2818



  CONCENTRATION IN SALT WATER (PPM CL)           22000.0000
  BACKGROUND CONCENTRATION (PPM CL)                145.0000
  INITIAL WIDTH OF TRANSITION ZONE (M )              3.5000

  DISPERSIVITY (M )                                    .5000
  INTERCEPTION COEFFICIENT                             .0800



  ENTER NEXT COMMAND
  ?
PR

  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
166.85
```

```
   SEMADAR 1  SALINITY/TIME RELATIONSHIPS

     DENSITY OF FRESH WATER                                1.0000
     DENSITY OF SALT WATER                                 1.0300

     AQUIFER POROSITY                                       .3300
     HORIZONTAL PERMEABILITY (M /DY)                      14.7000
     VERTICAL PERMEABILITY (M /DY)                        14.7000

     INITIAL INTERFACE ELEVATION (M )                    -30.7500
     DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )       15.5000
     FRACTIONAL CRITICAL RISE                               .4000
     CRITICAL RISE (M )                                    6.2000
     CRITICAL ELEVATION (M )                             -24.5500

     CONCENTRATION IN SALT WATER (PPM CL)              22000.0000
     BACKGROUND CONCENTRATION (PPM CL)                   145.0000
     INITIAL WIDTH OF TRANSITION ZONE (M )                 3.5000

     DISPERSIVITY (M )                                      .5000
     INTERCEPTION COEFFICIENT                               .0800



     MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)      166.8500
     MAXIMUM RELATIVE CONCENTRATION                         .0010
     MAXIMUM INTERFACE ELEVATION (M )                    -28.8973
     MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)          79.5689

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
   ?
348.


  PUMPING RATE (CU M /DY)          348.0000
  TIME TO REACH CMAX (DY)            6.8762

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
   ?
575.


  PUMPING RATE (CU M /DY)          575.0000
  TIME TO REACH CMAX (DY)            3.7256

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
   ?
0.


  ENTER NEXT COMMAND
   ?
PR


  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
   ?
210.56
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                1.0000
   DENSITY OF SALT WATER                                 1.0300

   AQUIFER POROSITY                                       .3300
   HORIZONTAL PERMEABILITY (M /DY)                      14.7000
   VERTICAL PERMEABILITY (M /DY)                        14.7000

   INITIAL INTERFACE ELEVATION (M )                    -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )       15.5000
   FRACTIONAL CRITICAL RISE                               .4000
   CRITICAL RISE (M )                                    6.2000
   CRITICAL ELEVATION (M )                             -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)              22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                   145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                 3.5000

   DISPERSIVITY (M )                                      .5000
   INTERCEPTION COEFFICIENT                               .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)      210.5600
   MAXIMUM RELATIVE CONCENTRATION                         .0030
   MAXIMUM INTERFACE ELEVATION (M )                    -28.0165
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)         117.4009

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
348.


  PUMPING RATE (CU M /DY)          348.0000
  TIME TO REACH CMAX (DY)           11.8100

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
575.


  PUMPING RATE (CU M /DY)          575.0000
  TIME TO REACH CMAX (DY)            5.9515

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.


  ENTER NEXT COMMAND
  ?
PR


  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
245.27
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                1.0000
   DENSITY OF SALT WATER                                 1.0300

   AQUIFER POROSITY                                       .3300
   HORIZONTAL PERMEABILITY (M /DY)                      14.7000
   VERTICAL PERMEABILITY (M /DY)                        14.7000

   INITIAL INTERFACE ELEVATION (M )                    -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )       15.5000
   FRACTIONAL CRITICAL RISE                               .4000
   CRITICAL RISE (M )                                    6.2000
   CRITICAL ELEVATION (M )                             -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)              22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                   145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                 3.5000

   DISPERSIVITY (M )                                      .5000
   INTERCEPTION COEFFICIENT                               .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)      245.2700
   MAXIMUM RELATIVE CONCENTRATION                         .0046
   MAXIMUM INTERFACE ELEVATION (M )                    -27.5565
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)         137.1553

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
348.


 PUMPING RATE (CU M /DY)          348.0000
 TIME TO REACH CMAX (DY)           15.0899

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
575.


 PUMPING RATE (CU M /DY)          575.0000
 TIME TO REACH CMAX (DY)            7.2666

 ENTER OPTIONAL PUMPING RATE (CU M /DY)
 ?
0.


 ENTER NEXT COMMAND
 ?
PR


 ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
 ?
363.55
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                1.0000
   DENSITY OF SALT WATER                                 1.0300

   AQUIFER POROSITY                                       .3300
   HORIZONTAL PERMEABILITY (M /DY)                      14.7000
   VERTICAL PERMEABILITY (M /DY)                        14.7000

   INITIAL INTERFACE ELEVATION (M )                    -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )       15.5000
   FRACTIONAL CRITICAL RISE                               .4000
   CRITICAL RISE (M )                                    6.2000
   CRITICAL ELEVATION (M )                             -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)              22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                   145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                 3.5000

   DISPERSIVITY (M )                                      .5000
   INTERCEPTION COEFFICIENT                               .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)      363.5500
   MAXIMUM RELATIVE CONCENTRATION                         .0100
   MAXIMUM INTERFACE ELEVATION (M )                    -26.3880
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)         187.3442

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
348.


  PUMPING RATE (CU M /DY)          348.0000
  TIME TO REACH CMAX (DY)           27.0509

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
575.


  PUMPING RATE (CU M /DY)          575.0000
  TIME TO REACH CMAX (DY)           11.2107

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.


  ENTER NEXT COMMAND
  ?
PR


  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
582.1
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                  1.0000
   DENSITY OF SALT WATER                                   1.0300

   AQUIFER POROSITY                                         .3300
   HORIZONTAL PERMEABILITY (M /DY)                        14.7000
   VERTICAL PERMEABILITY (M /DY)                          14.7000

   INITIAL INTERFACE ELEVATION (M )                      -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )         15.5000
   FRACTIONAL CRITICAL RISE                                 .4000
   CRITICAL RISE (M )                                     6.2000
   CRITICAL ELEVATION (M )                              -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)                22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                     145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                   3.5000

   DISPERSIVITY (M )                                        .5000
   INTERCEPTION COEFFICIENT                                 .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)        582.1000
   MAXIMUM RELATIVE CONCENTRATION                           .0200
   MAXIMUM INTERFACE ELEVATION (M )                     -24.5511
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)          266.2366

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
348.

  PUMPING RATE (CU M /DY)          348.0000
  TIME TO REACH CMAX (DY)           75.5346

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
575.

  PUMPING RATE (CU M /DY)          575.0000
  TIME TO REACH CMAX (DY)           20.0023

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.

  ENTER NEXT COMMAND
  ?
PR

  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
800.65
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                              1.0000
   DENSITY OF SALT WATER                               1.0300

   AQUIFER POROSITY                                     .3300
   HORIZONTAL PERMEABILITY (M /DY)                    14.7000
   VERTICAL PERMEABILITY (M /DY)                      14.7000

   INITIAL INTERFACE ELEVATION (M )                  -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )     15.5000
   FRACTIONAL CRITICAL RISE                             .4000
   CRITICAL RISE (M )                                  6.2000
   CRITICAL ELEVATION (M )                           -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)            22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                 145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )               3.5000

   DISPERSIVITY (M )                                    .5000
   INTERCEPTION COEFFICIENT                             .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)    800.6500
   MAXIMUM RELATIVE CONCENTRATION                       .0300
   MAXIMUM INTERFACE ELEVATION (M )                  -22.2571*
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)       364.7604

 * NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW
         THE CRITICAL ELEVATION OF    -24.5500 M
 (MAXIMUM CONCENTRATIONS IN PUMPED WATER LESS THAN    582.10 PPM CL)


  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
575.

  PUMPING RATE (CU M /DY)          575.0000
  TIME TO REACH CMAX (DY)           40.2467

  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.

  ENTER NEXT COMMAND
  ?
PR

  ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED WATER (PPM CL)
  ?
1019.2
```

```
 SEMADAR 1  SALINITY/TIME RELATIONSHIPS

   DENSITY OF FRESH WATER                                  1.0000
   DENSITY OF SALT WATER                                   1.0300

   AQUIFER POROSITY                                         .3300
   HORIZONTAL PERMEABILITY (M /DY)                        14.7000
   VERTICAL PERMEABILITY (M /DY)                          14.7000

   INITIAL INTERFACE ELEVATION (M )                      -30.7500
   DISTANCE FROM BOTTOM OF WELL TO INTERFACE (M )         15.5000
   FRACTIONAL CRITICAL RISE                                 .4000
   CRITICAL RISE (M )                                      6.2000
   CRITICAL ELEVATION (M )                               -24.5500

   CONCENTRATION IN SALT WATER (PPM CL)                22000.0000
   BACKGROUND CONCENTRATION (PPM CL)                     145.0000
   INITIAL WIDTH OF TRANSITION ZONE (M )                   3.5000

   DISPERSIVITY (M )                                        .5000
   INTERCEPTION COEFFICIENT                                 .0800



   MAXIMUM CONCENTRATION IN PUMPED WATER (PPM CL)      1019.2000
   MAXIMUM RELATIVE CONCENTRATION                           .0400
   MAXIMUM INTERFACE ELEVATION (M )                    -13.1997*
   MAXIMUM PERMISSIBLE PUMPING RATE (CU M /DY)         753.7639

 * NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED TO THE ZONE BELOW
         THE CRITICAL ELEVATION OF    -24.5500 M
 (MAXIMUM CONCENTRATIONS IN PUMPED WATER LESS THAN    582.10 PPM CL)


  ENTER OPTIONAL PUMPING RATE (CU M /DY)
  ?
0.


  ENTER NEXT COMMAND
  ?
DN
Stop - Program terminated.

C>
```

APPENDIX B

Description of Program UPCONE

Program UPCONE has been written in an unextended Fortran computer code in an effort to make the program transportable between computer systems. The code consists of a main program and two function subroutines. The program has been documented internally through the liberal use of comment statements.

The main program is divided into three sections. Section I provides for the "Basic Input Data" as described in Section III of this report. The numerical evaluation of interface elevations, concentrations, and maximum pumping rates is accomplished in Section II of the main program which contains the computational algorithms for Case I and Case II types of problems. Section III provides for problem redefinition and control of execution under the "Edit" mode.

Two function subroutines are required to calculate the concentration distribution across the transition zone or to evaluate the maximum pumping rate for a specified salinity of the pumped water. Function ERFC (Z) is a rational approximation of the complimentary error function of the argument Z.

Function IERFC (X, Z, ZMIN, ZMAX) uses a *regula falsi* root finding technique to find the argument, Z, for a specified value of the complimentary error function, X. The last two arguments, ZMIN and ZMAX, define the lower and upper limits of the initial search interval.

APPENDIX C

Listing of Program UPCONE

```
C     PROGRAM UPCONE                                                    UPCN001
C     VERSION 1.0                                                       UPCN002
C     JAN WAGNER                                                        UPCN003
C     SCHOOL OF CHEMICAL ENGINEERING                                    UPCN004
C     OKLAHOMA STATE UNIVERSITY                                         UPCN005
C     STILLWATER, OK  74078                                             UPCN006
C     TELEPHONE (405) 624-5280                                          UPCN007
C                                                                       UPCN008
C     JANUARY, 1982                                                     UPCN009
C                                                                       UPCN010
      DIMENSION A(30),C(11),CW(51),E(11),EW(51),IC(20),KFLG(11),        UPCN011
     1R(26),T(51),Z(26,26)                                              UPCN012
      REAL*4 KX,KZ                                                      UPCN013
      DATA KHAR1,KHAR2,NY/' ','*','Y'/                                  UPCN014
      DATA IC/'FD','PD','KX','KZ','ZO','DT','CR','CO','DI','WT','IC',  UPCN015
     1'OB','TC','RC','QP','LI','EL','PR','NP','DN'/                    UPCN016
C                                                                       UPCN017
C     READ DEVICE: NI=1       WRITE DEVICE: NO=1                        UPCN018
      NI = 1                                                            UPCN019
      NO = 1                                                            UPCN020
C                                                                       UPCN021
C     MAXIMUM NUMBERS OF OBSERVATION POINTS FOR TIME AND RADIUS         UPCN022
C     HAVE BEEN SET AT 50 AND 25, RESPECTIVELY                          UPCN023
C      DIMENSION CW(M),EW(M),R(N),T(N),Z(M,N)                           UPCN024
C         -- WHERE M=MAXTIM+1 AND N=MAXPTS+1                            UPCN025
      MAXTIM = 50                                                       UPCN026
      MAXPTS = 25                                                       UPCN027
C                                                                       UPCN028
C     INITIALIZE PROGRAM FLOW PARAMETERS                                UPCN029
    1 IEDIT = 1                                                         UPCN030
      KCON = 1                                                          UPCN031
      KELE = 1                                                          UPCN032
C                                                                       UPCN033
C ***** SECTION I -- BASIC INPUT DATA                                   UPCN034
C                                                                       UPCN035
C     READ TITLE                                                        UPCN036
      WRITE(NO,5)                                                       UPCN037
    5 FORMAT('1',3X,'ENTER TITLE',/,3X,'?')                             UPCN038
      READ(NI,10) (A(I),I=1,30)                                         UPCN039
   10 FORMAT(30A2)                                                      UPCN040
C                                                                       UPCN041
C     DEFINE UNITS                                                      UPCN042
      WRITE(NO,15)                                                      UPCN043
   15 FORMAT(3X,'ENTER UNITS FOR LENGTH (2 CHARACTERS)',/,3X,'?')       UPCN044
      READ(NI,20) IL                                                    UPCN045
   20 FORMAT(A2)                                                        UPCN046
      WRITE(NO,25)                                                      UPCN047
   25 FORMAT(3X,'ENTER UNITS FOR TIME (2 CHARACTERS)',/,3X,'?')         UPCN048
      READ(NI,20) ITU                                                   UPCN049
C                                                                       UPCN050
C     FLUID DENSITIES                                                   UPCN051
   29 WRITE(NO,30)                                                      UPCN052
   30 FORMAT(3X,'ENTER FRESH-WATER AND SALT-WATER DENSITIES',/,3X,'?,?') UPCN053
      READ(NI,35) RHOF,RHOS                                             UPCN054
   35 FORMAT(2F10.0)                                                    UPCN055
   40 IF(RHOS.GT.0.0) GO TO 55                                          UPCN056
      WRITE(NO,45)                                                      UPCN057
   45 FORMAT(3X,'SALT-WATER DENSITY MUST BE GREATER THAN ZERO',         UPCN058
     1' -- REENTER',/,3X,'?')                                           UPCN059
      READ(NI,50) RHOS                                                  UPCN060
   50 FORMAT(F10.0)                                                     UPCN061
      GO TO 40                                                          UPCN062
   55 IF(RHOF.GT.0.0.AND.RHOF.LT.RHOS) GO TO 69                         UPCN063
      WRITE(NO,60)                                                      UPCN064
   60 FORMAT(3X,'FRESH-WATER DENSITY MUST BE GREATER THAN ZERO',        UPCN065
     1/,6X,'AND LESS THAN SALT-WATER DENSITY -- REENTER',/,3X,'?')      UPCN066
      READ(NI,50) RHOF                                                  UPCN067
      GO TO 55                                                          UPCN068
   69 GO TO (70,700), IEDIT                                             UPCN069
C                                                                       UPCN070
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C     POROSITY                                                          UPCN071
   70 WRITE(NO,75)                                                      UPCN072
   75 FORMAT(3X,'ENTER AQUIFER POROSITY',/,3X,'?')                      UPCN073
   80 READ(NI,50) PO                                                    UPCN074
      IF(PO.GT.0.0.AND.PO.LT.1.0) GO TO 89                              UPCN075
      WRITE(NO,85)                                                      UPCN076
   85 FORMAT(3X,'POROSITY MUST BE GREATER THAN ZERO AND LESS THAN',     UPCN077
     1' ONE -- REENTER',/,3X,'?')                                       UPCN078
      GO TO 80                                                          UPCN079
   89 GO TO (90,700), IEDIT                                             UPCN080
C                                                                       UPCN081
C     HORIZONTAL PERMEABILITY                                           UPCN082
   90 WRITE(NO,95) IL,ITU                                               UPCN083
   95 FORMAT(3X,'ENTER HORIZONTAL PERMEABILITY (',A2,'/',A2,') ',       UPCN084
     1/,3X,'?')                                                         UPCN085
  100 READ(NI,50) KX                                                    UPCN086
      IF(KX.GT.0.0) GO TO 110                                           UPCN087
      WRITE(NO,105)                                                     UPCN088
  105 FORMAT(3X,'HORIZONTAL PERMEABILITY MUST BE GREATER THAN',         UPCN089
     1' ZERO -- REENTER',/,3X,'?')                                      UPCN090
      GO TO 100                                                         UPCN091
  110 GO TO (111,700), IEDIT                                            UPCN092
C                                                                       UPCN093
C     VERTICAL PERMEABILITY                                             UPCN094
  111 WRITE(NO,112) IL,ITU                                              UPCN095
  112 FORMAT(3X,'ENTER VERTICAL PERMEABILITY (',1A2,'/',A2,') ',        UPCN096
     1/,3X,'?')                                                         UPCN097
  114 READ(NI,50) KZ                                                    UPCN098
      IF(KZ.GT.0.0) GO TO 119                                           UPCN099
      WRITE(NO,115)                                                     UPCN100
  115 FORMAT(3X,'VERTICAL PERMEABILITY MUST BE GREATER THAN ZERO',      UPCN101
     1' -- REENTER',/,3X,'?')                                           UPCN102
      GO TO 114                                                         UPCN103
  119 GO TO (120,700), IEDIT                                            UPCN104
C                                                                       UPCN105
C     INITIAL INTERFACE ELEVATION                                       UPCN106
  120 WRITE(NO,125) IL                                                  UPCN107
  125 FORMAT(3X,'ENTER INITIAL INTERFACE ELEVATION (',A2,')',/,3X,'?')  UPCN108
      READ(NI,50) ZO                                                    UPCN109
      GO TO (129,700), IEDIT                                            UPCN110
C                                                                       UPCN111
C     DISTANCE FROM BOTTOM OF WELL TO INITIAL INTERFACE                 UPCN112
  129 WRITE(NO,130) IL                                                  UPCN113
  130 FORMAT(3X,'ENTER DISTANCE FROM BOTTOM OF WELL TO INITIAL ',       UPCN114
     1'INTERFACE (',A2,') ',/,3X,'?')                                   UPCN115
  135 READ(NI,50) D                                                     UPCN116
      IF(D.GT.0.0) GO TO 144                                            UPCN117
      WRITE(NO,140)                                                     UPCN118
  140 FORMAT(3X,'DISTANCE MUST BE GREATER THAN ZERO -- REENTER',/,      UPCN119
     13X,'?')                                                           UPCN120
      GO TO 135                                                         UPCN121
  144 GO TO (145,700), IEDIT                                            UPCN122
C                                                                       UPCN123
C     FRACTIONAL CRITICAL RISE                                          UPCN124
  145 WRITE(NO,150)                                                     UPCN125
  150 FORMAT(3X,'ENTER FRACTIONAL CRITICAL RISE',/,3X,'?')              UPCN126
  155 READ(NI,50) THETA                                                 UPCN127
      IF(THETA.GT.0.0.AND.THETA.LT 1.0) GO TO 164                       UPCN128
      WRITE(NO,160)                                                     UPCN129
  160 FORMAT(3X,'FRACTION MUST BE GREATER THAN ZERO AND LESS THAN',     UPCN130
     1' ONE -- REENTER',/,3X,'?')                                       UPCN131
      GO TO 155                                                         UPCN132
  164 XCR = THETA*D                                                     UPCN133
      ZCR = XCR + ZO                                                    UPCN134
C     MAXIMUM STEADY-STATE PUMPING RATE                                 UPCN135
      QMAXSS = 6.283185*((RHOS-RHOF)/RHOF)*KX*D*XCR                     UPCN136
  165 GO TO (169,700), IEDIT                                            UPCN137
C                                                                       UPCN138
C                                                                       UPCN139
C     DATA FOR CONCENTRATION CALCULATIONS                               UPCN140
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  169 WRITE(NO,170)                                                    UPCN141
  170 FORMAT('0',2X,'CONCENTRATION CALCULATIONS ? (Y/N)')              UPCN142
      READ(NI,175) ICON                                                UPCN143
  175 FORMAT(A1)                                                       UPCN144
C                                                                      UPCN145
      IF(ICON.NE.NY) GO TO 275                                         UPCN146
C                                                                      UPCN147
  176 KCON = 2                                                         UPCN148
      WRITE(NO,180)                                                    UPCN149
  180 FORMAT(3X,'ENTER UNITS FOR CONCENTRATION (6 CHARACTERS)',        UPCN150
     1/,3X,'?')                                                        UPCN151
      READ(NI,185) IM1,IM2,IM3                                         UPCN152
  185 FORMAT(3A2)                                                      UPCN153
C                                                                      UPCN154
C     SALT-WATER AND BACKGROUND CONCENTRATIONS                         UPCN155
  188 IF(KCON.EQ.2) GO TO 189                                          UPCN156
      WRITE(NO,180)                                                    UPCN157
      READ(NI,185) IM1,IM2,IM3                                         UPCN158
  189 WRITE(NO,190) IM1,IM2,IM3                                        UPCN159
  190 FORMAT(3X,'ENTER SALT-WATER AND BACKGROUND CONCENTRATIONS (',    UPCN160
     13A2,') ',/,3X,'?,?')                                            UPCN161
      READ(NI,35) CO,CB                                                UPCN162
  195 IF(CO.GE.1.0) GO TO 205                                          UPCN163
      WRITE(NO,200)                                                    UPCN164
  200 FORMAT(3X,'SALT-WATER CONCENTRATION MUST BE GREATER THAN',       UPCN165
     1' OR EQUAL TO ONE -- REENTER',/,3X,'?')                          UPCN166
      READ(NI,50) CO                                                   UPCN167
      GO TO 195                                                        UPCN168
  205 IF(CB.GE.0.0.AND.CB.LT.CO) GO TO 214                             UPCN169
      WRITE(NO,210)                                                    UPCN170
  210 FORMAT(3X,'BACKGROUND CONCENTRATION MUST BE GREATER THAN',       UPCN171
     1' OR EQUAL TO ZERO',/,6X,'AND LESS THAN SALT-WATER',             UPCN172
     2' CONCENTRATION -- REENTER',/,3X,'?')                            UPCN173
      READ(NI,50) CB                                                   UPCN174
      GO TO 205                                                        UPCN175
  214 GO TO (215,700,215,215), IEDIT                                   UPCN176
C                                                                      UPCN177
C     DISPERSIVITY                                                     UPCN178
  215 WRITE(NO,220) IL                                                 UPCN179
  220 FORMAT(3X,'ENTER DISPERSIVITY (',A2,') ',/,3X,'?')               UPCN180
  225 READ(NI,50) DM                                                   UPCN181
      IF(DM.GT.0.0) GO TO 234                                          UPCN182
      WRITE(NO,230)                                                    UPCN183
  230 FORMAT(3X,'DISPERSIVITY MUST BE GREATER THAN ZERO -- REENTER',   UPCN184
     1/,3X,'?')                                                        UPCN185
      GO TO 225                                                        UPCN186
  234 GO TO (235,700,235,235), IEDIT                                   UPCN187
C                                                                      UPCN188
C     INITIAL WIDTH OF TRANSITION ZONE                                 UPCN189
  235 WRITE(NO,240) IL                                                 UPCN190
  240 FORMAT(3X,'ENTER INITIAL WIDTH OF TRANSITION ZONE (',A2,') ',    UPCN191
     1/,3X,'?')                                                        UPCN192
  245 READ(NI,50) SO2                                                  UPCN193
      IF(SO2.GE.0.0) GO TO 254                                         UPCN194
      WRITE(NO,250)                                                    UPCN195
  250 FORMAT(3X,'INITIAL WIDTH MUST BE GREATER THAN ZERO -- REENTER',  UPCN196
     1/,3X,'?')                                                        UPCN197
      GO TO 245                                                        UPCN198
  254 GO TO (255,700,255,255), IEDIT                                   UPCN199
C                                                                      UPCN200
C     INTERCEPTION COEFFICIENT                                         UPCN201
  255 WRITE(NO,260)                                                    UPCN202
  260 FORMAT(3X,'ENTER INTERCEPTION COEFFICIENT',/,3X,'?')             UPCN203
  265 READ(NI,50) PHI                                                  UPCN204
      IF(PHI.GT.0.0.AND.PHI.LT.1.0) GO TO 274                          UPCN205
      WRITE(NO,270)                                                    UPCN206
  270 FORMAT(3X,'COEFFICIENT MUST BE GREATER THAN ZERO AND LESS THAN', UPCN207
     1' ONE -- REENTER',/,3X,'?')                                      UPCN208
      GO TO 265                                                        UPCN209
  274 GO TO (275,700,275,510), IEDIT                                   UPCN210
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C                                                                       UPCN211
C     LIST PROBLEM DEFINITION                                           UPCN212
C                                                                       UPCN213
  275 CONTINUE                                                          UPCN214
      WRITE(NO,280) (A(I),I=1,30), RHOF,RHOS,PO,IL,ITU,KX,IL,ITU,KZ     UPCN215
  280 FORMAT('1',3X,30A2,//,                                            UPCN216
     16X,'DENSITY OF FRESH WATER',25X,F10.4,/,                          UPCN217
     26X,'DENSITY OF SALT WATER',26X,F10.4,//,                          UPCN218
     36X,'AQUIFER POROSITY',31X,F10.4,/,                                UPCN219
     46X,'HORIZONTAL PERMEABILITY (',A2,'/',A2,') ',15X,F10.4,/,        UPCN220
     56X,'VERTICAL PERMEABILITY (',A2,'/',A2,') ',17X,F10.4)            UPCN221
      WRITE(NO,281)IL, ZO,IL,D,THETA,IL,XCR,IL,ZCR                      UPCN222
  281 FORMAT('0',                                                       UPCN223
     15X,'INITIAL INTERFACE ELEVATION (',A2,') ',14X,F10.4,/,           UPCN224
     26X,'DISTANCE FROM BOTTOM OF WELL TO INTERFACE (',A2,') ',F10.4,/, UPCN225
     36X,'FRACTIONAL CRITICAL RISE ',22X,F10.4,/,                       UPCN226
     46X,'CRITICAL RISE (',A2,') ',28X,F10.4,/,                         UPCN227
     56X,'CRITICAL ELEVATION (',A2,') ',23X,F10.4)                      UPCN228
C                                                                       UPCN229
      WRITE(NO,285) IL,ITU,QMAXSS                                       UPCN230
  285 FORMAT('0',5X,'MAXIMUM STEADY-STATE PUMPING RATE (CU ',A2,'/',    UPCN231
     1A2,')   ',F10.4,/,'0',/,'0')                                      UPCN232
C                                                                       UPCN233
      IF(ICON.NE.NY.OR.KCON.EQ.1) GO TO 700                             UPCN234
C                                                                       UPCN235
      WRITE(NO,290) IM1,IM2,IM3,CO,IM1,IM2,IM3,CB,IL,SO2,IL,DM,PHI      UPCN236
  290 FORMAT('0',5X,'CONCENTRATION IN SALT WATER (',3A2,') ',10X,F10.4,/ UPCN237
     16X,'BACKGROUND CONCENTRATION (',3A2,') ',13X,F10.4,/,             UPCN238
     26X,'INITIAL WIDTH OF TRANSITION ZONE (',A2,') ',9X,F10.4,//,      UPCN239
     36X,'DISPERSIVITY (',A2,') ',29X,F10.4,/,                          UPCN240
     46X,'INTERCEPTION COEFFICIENT',23X,F10.4,/,'0',/,'0')              UPCN241
C                                                                       UPCN242
      GO TO (700,700,301), IEDIT                                        UPCN243
C                                                                       UPCN244
C                                                                       UPCN245
C ***** SECTION II -- NUMERICAL EVALUATION OF INTERFACE ELEVATIONS      UPCN246
C                                                                       UPCN247
C                                                                       UPCN248
C     CASE I PROBLEMS -- EVALUATE INTERFACE ELEVATIONS AND CONCENTRATION UPCN249
C                                                                       UPCN250
C                                                                       UPCN251
  300 CONTINUE                                                          UPCN252
      IEDIT = 3                                                         UPCN253
                                                                        UPCN254
C     PARAMETERS FOR INTERFACE ELEVATION CALCULATIONS                   UPCN255
  301 IF(KELE.EQ.2) GO TO 329                                           UPCN256
      KELE = 2                                                          UPCN257
C                                                                       UPCN258
C     PUMPING RATE AND PERIOD                                           UPCN259
  302 WRITE(NO,305) IL,ITU,ITU                                          UPCN260
  305 FORMAT('0',2X,'ENTER PUMPING RATE (CU ',A2,'/',A2,') AND',        UPCN261
     1' PERIOD (',A2,')',/,3X,'?,?')                                    UPCN262
      READ(NI,35) Q,TPUMP                                               UPCN263
  306 IF(Q.GT.0.0) GO TO 308                                            UPCN264
      WRITE(NO,307)                                                     UPCN265
  307 FORMAT(3X,'PUMPING RATE MUST BE GREATER THAN ZERO -- REENTER',    UPCN266
     1/,3X,'?')                                                         UPCN267
      READ(NI,50) Q                                                     UPCN268
      GO TO 306                                                         UPCN269
  308 IF(TPUMP.GT.0.0) GO TO 310                                        UPCN270
      WRITE(NO,309)                                                     UPCN271
  309 FORMAT(3X,'PUMPING PERIOD MUST BE GREATER THAN ZERO -- REENTER',  UPCN272
     1/,3X,'?')                                                         UPCN273
      READ(NI,50) TPUMP                                                 UPCN274
      GO TO 308                                                         UPCN275
  310 GO TO (700,700,312), IEDIT                                        UPCN276
C                                                                       UPCN277
C     COORDINATES OF OBSERVATION POINTS -- TIME AND RADIUS              UPCN278
  311 IEDIT = 1                                                         UPCN279
  312 WRITE(NO,313) ITU                                                 UPCN280
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  313 FORMAT(3X,'ENTER TFIRST, TLAST, DELTAT (',A2,') ',/,3X,'?,?,?')     UPCN281
      READ(NI,315) TF,TL,DELT                                             UPCN282
  315 FORMAT(3F10.0)                                                      UPCN283
      DELT = ABS(DELT)                                                    UPCN284
  316 IF(TF.GE.0.0.AND.DELT.LE.1.0E-06) GO TO 320                         UPCN285
      IF(TF.GE.0.0) GO TO 318                                             UPCN286
      WRITE(NO,317)                                                       UPCN287
  317 FORMAT(3X,'TFIRST MUST NOT BE LESS THAN ZERO -- REENTER',           UPCN288
     1/,3X,'?')                                                           UPCN289
      READ(NI,50) TF                                                      UPCN290
      GO TO 316                                                           UPCN291
  318 IF(TL.GE.0.0) GO TO 321                                             UPCN292
      WRITE(NO,319)                                                       UPCN293
  319 FORMAT(3X,'TLAST MUST NOT BE LESS THAN ZERO -- REENTER',            UPCN294
     1/,3X,'?')                                                           UPCN295
      READ(NI,50) TL                                                      UPCN296
      GO TO 318                                                           UPCN297
  320 TL = TF                                                             UPCN298
  321 GO TO (322,700,322), IEDIT                                          UPCN299
C                                                                         UPCN300
  322 WRITE(NO,323) IL                                                    UPCN301
  323 FORMAT(3X,'ENTER RFIRST, RLAST, DELTAR (',A2,') ',/,3X,'?,?,?')     UPCN302
      READ(NI,325) RF,RL,DELR                                             UPCN303
  325 FORMAT(3F10.0)                                                      UPCN304
      DELR = ABS(DELR)                                                    UPCN305
      GO TO (700,700,329), IEDIT                                          UPCN306
C                                                                         UPCN307
  329 WRITE(NO,330) IL,ITU,Q,ITU,TPUMP,TF,TL,DELT,RF,RL,DELR              UPCN308
  330 FORMAT('O',5X,'PUMPING RATE (CU ',A2,'/',A2,') ',23X,F10.4,/,       UPCN309
     16X,'PUMPING PERIOD (',A2,') ',27X,F10.4,//,                         UPCN310
     26X,'TFIRST =',F10.4,3X,'TLAST =',F10.4,3X,'DELTAT =',F10.4,/,       UPCN311
     36X,'RFIRST =',F10.4,3X,'RLAST =',F10.4,3X,'DELTAR =',F10.4)         UPCN312
      IF(Q.LE.QMAXSS) GO TO 332                                           UPCN313
      TCR = ((2.*PO*D)/(((RHOS-RHOF)/RHOF)*KZ))*((1./(1.-QMAXSS/Q))-1.)   UPCN314
      WRITE(NO,331) TCR,ITU                                               UPCN315
  331 FORMAT('O',2X,'NOTE:  INTERFACE WILL RISE TO CRITICAL ELEVATION',   UPCN316
     1' IN',F10.2,A3)                                                     UPCN317
  332 WRITE(NO,333)                                                       UPCN318
  333 FORMAT('O',2X,'CONTINUE ? (Y/N)')                                   UPCN319
      READ(NI,175) JFLOW                                                  UPCN320
      IF(JFLOW.NE.NY) GO TO 700                                           UPCN321
C                                                                         UPCN322
C     RADIUS COORDINATES                                                  UPCN323
  335 CONTINUE                                                            UPCN324
      IR = 1                                                              UPCN325
      R(IR) = RF                                                          UPCN326
      IF(DELR.LE.1.0E-06) GO TO 345                                       UPCN327
      DIF = RL - RF                                                       UPCN328
      IF(ABS(DIF).LE.1.0E-06) GO TO 345                                   UPCN329
      IF(DIF.LE.0.0) DELR = -DELR                                         UPCN330
      NPTS = DIF/DELR                                                     UPCN331
      REM = DIF - DELR*FLOAT(NPTS)                                        UPCN332
      TOL = 1.0E-06*ABS(DIF)                                              UPCN333
      NPTS = NPTS + 1                                                     UPCN334
      IF(NPTS.LE.MAXPTS) GO TO 337                                        UPCN335
      WRITE(NO,336) NPTS,MAXPTS                                           UPCN336
  336 FORMAT(3X,I3,' RADIUS OBSERVATION POINTS EXCEED MAXIMUM OF',I4)     UPCN337
      GO TO 700                                                           UPCN338
  337 CONTINUE                                                            UPCN339
      DO 340  IR=2,NPTS                                                   UPCN340
       R(IR) = R(IR-1) + DELR                                             UPCN341
  340 CONTINUE                                                            UPCN342
      IR = NPTS                                                           UPCN343
      IF(ABS(REM).LT.TOL) GO TO 345                                       UPCN344
      IR = IR + 1                                                         UPCN345
      R(IR) = RL                                                          UPCN346
  345 CONTINUE                                                            UPCN347
C                                                                         UPCN348
C     TIME COORDINATES                                                    UPCN349
      IT = 1                                                              UPCN350
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      T(IT) = TF                                                        UPCN351
      IF(DELT.LE.1.0E-06) GO TO 355                                     UPCN352
      DIF = TL - TF                                                     UPCN353
      IF(ABS(DIF).LE.1.0E-06) GO TO 355                                 UPCN354
      IF(DIF.LE.0.0) DELT = -DELT                                       UPCN355
      NPTS = DIF/DELT                                                   UPCN356
      REM = DIF - DELT*FLOAT(NPTS)                                      UPCN357
      TOL = 1.0E-06*ABS(DIF)                                            UPCN358
      NPTS = NPTS + 1                                                   UPCN359
      IF(NPTS.LE.MAXTIM) GO TO 347                                      UPCN360
      WRITE(NO,346) NPTS,MAXTIM                                         UPCN361
  346 FORMAT(3X,I3,' TIME OBSERVATION POINTS EXCEED MAXIMUM OF',I4)      UPCN362
      GO TO 700                                                         UPCN363
  347 CONTINUE                                                          UPCN364
      DO 350  IT=2,NPTS                                                 UPCN365
       T(IT) = T(IT-1) + DELT                                           UPCN366
  350 CONTINUE                                                          UPCN367
      IT = NPTS                                                         UPCN368
      IF(ABS(REM).LT.TOL) GO TO 355                                     UPCN369
      IT = IT + 1                                                       UPCN370
      T(IT) = TL                                                        UPCN371
  355 CONTINUE                                                          UPCN372
      TMAX = TL                                                         UPCN373
      IF(TF.GT.TL) TMAX=TF                                              UPCN374
C                                                                       UPCN375
C                                                                       UPCN376
      COEF = Q/(6.2832*((RHOS-RHOF)/RHOF)*KX*D)                         UPCN377
      CONR = SQRT(KZ/KX)/D                                              UPCN378
      CONT = ((RHOS-RHOF)/RHOF)*KZ/(2.0*PO*D)                           UPCN379
      DO 365  I=1,IT                                                    UPCN380
       TAU = CONT*T(I)                                                  UPCN381
       TAU1 = CONT*(T(I)-TPUMP)                                         UPCN382
       IF(T(I).LE.TPUMP) TAU1=0.0                                       UPCN383
       XRO = COEF*(1.0/(1.0+TAU1) - 1.0/(1.0+TAU))                      UPCN384
       ZRO = XRO + ZO                                                   UPCN385
       DO 360  J=1,IR                                                   UPCN386
          RDIM = CONR*R(J)                                              UPCN387
          Z(I,J) = COEF*((1.0/SQRT((1.0+TAU1)**2 + RDIM**2))            UPCN388
     1                                                                  UPCN389
  360  CONTINUE                                                         UPCN390
      IF(ZRO.GT.ZCR) IPR=2                                              UPCN391
  365 CONTINUE                                                          UPCN392
  370 IT = I                                                            UPCN393
C                                                                       UPCN394
C     PRINT INTERFACE ELEVATIONS                                        UPCN395
      WRITE(NO,375) Q,IL,ITU,TPUMP,ITU,IL,IL                            UPCN396
  375 FORMAT('1',18X,'PUMPING RATE:',F12.2,' CU ',A2,'/',A2,' FOR',     UPCN397
     1F10.2,A3,/,'0',18X,'INTERFACE ELEVATIONS (',A2,') ',//,           UPCN398
     23X,' *',/,3X,'    * R (',A2,')')                                 UPCN399
      LIM1 = 1                                                          UPCN400
      LIM2 = 7                                                          UPCN401
  380 IF(LIM2.GT.IR) LIM2=IR                                            UPCN402
      WRITE(NO,385) (R(L),L=LIM1,LIM2)                                  UPCN403
  385 FORMAT(3X,'      *     ',7F9.2)                                   UPCN404
      WRITE(NO,390) ITU                                                 UPCN405
  390 FORMAT(3X,'T (',A2,') *',/,12X,'*')                               UPCN406
      DO 400 I=1,IT                                                     UPCN407
       DO 393  L=LIM1,LIM2                                              UPCN408
       KFLG(L) = KHAR1                                                  UPCN409
       IF(Z(I,L).GT.ZCR) KFLG(L)=KHAR2                                  UPCN410
  393  CONTINUE                                                         UPCN411
       WRITE(NO,395) T(I),(Z(I,L),KFLG(L),L=LIM1,LIM2)                  UPCN412
  395  FORMAT(5X,F8.2,1X,7(F8.2,A1))                                    UPCN413
  40AX.GE.TCR)                                                          UPCN414
     1                                                                  UPCN415
  405 FORMAT('0',2X,'* NOTE: CRITICAL ELEVATION OF',F8.2,A3,            UPCN416
     1' EXCEEDED AT R=0 AND T=',F8.2,A3)                                UPCN417
      IF(LIM2.EQ.IR) GO TO 415                                          UPCN418
      LIM1 = LIM1 + 7                                                   UPCN419
      LIM2 = LIM2 + 7                                                   UPCN420
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      WRITE(NO,410) IL,IL                                                  UPCN421
  410 FORMAT('1',18X,'INTERFACE ELEVATIONS (',A2,') (CONTINUED)',//,       UPCN422
     13X,' =',/,3X,'   * R (',A2,')')                                      UPCN423
      GO TO 380                                                            UPCN424
  415 CONTINUE                                                             UPCN425
      IF(ICON.NE.NY) GO TO 700                                             UPCN426
C                                                                          UPCN427
C     CONCENTRATION PROFILES                                               UPCN428
      SO = SO2/2.0                                                         UPCN429
      E(1) = 0.0                                                           UPCN430
      C(1) = CB                                                            UPCN431
      DO 420  K=2,11                                                       UPCN432
       E(K) = E(K-1) + 0.1                                                 UPCN433
       C(K) = E(K)*(CO-CB) + CB                                            UPCN434
  420 CONTINUE                                                             UPCN435
C                                                                          UPCN436
UP = CONT*TPUMP                                                            UPCN437
       XBAR1 = COEF*(1.0-1.0/(1.0+TAUP))                                   UPCN438
       TAU1 = CONT*(T(I)-TPUMP)                                            UPCN439
       XTOT = COEF*((1.0/(1.0+TAU1)) - (1.0/(1.0+TAU)))                    UPCN440
       XBAR2 = XBAR1 - XTOT                                                UPCN441
       XBAR = XBAR1 + XBAR2                                                UPCN442
       GO TO 440                                                           UPCN443
  435  XTOT = COEF*(1.0 - 1.0/(1.0+TAU))                                   UPCN444
       XBAR = XTOT                                                         UPCN445
  440  CONTINUE                                                            UPCN446
       S1 = SQRT(SO**2 + 2.0*DM*XBAR)                                      UPCN447
       ARG = 10.0                                                          UPCN448
       IF(S1.GT.0.0) ARG = (XCR-XTOT)/(1.414214*S1)                        UPCN449
       EZCR = 0.5*ERFC(ARG)                                                UPCN450
       EW(I) = 0.5*EZCR*PHI                                                UPCN451
       CW(I) = EW(I)*(CO-CB) + CB                                          UPCN452
       Z(I,1) = XTOT + 2.5*S1 + ZO                                         UPCN453
       Z(I,11) = XTOT - 2.5*S1 + ZO                                        UPCN454
       XLIM1 = 2.0                                                         UPCN455
       XLIM2 = 0.0                                                         UPCN456
       DO 445  K=2,5                                                       UPCN457
          CERF = 2.0*E(K)                                                  UPCN458
          CALL IERFC(CERF,ARG,XLIM1,XLIM2)                                 UPCN459
          DIST = 1.41421*S1*ARG                                            UPCN460
          Z(I,K) = XTOT + DIST + ZO                                        UPCN461
          L = 12-K                                                         UPCN462
          Z(I,L) = XTOT - DIST + ZO                                        UPCN463
  445  CONTINUE                                                            UPCN464
       Z(I,6) = XTOT + ZO                                                  UPCN465
  446 CONTINUE                                                             UPCN466
      LIM1 = 1                                                             UPCN467
      LIM2 = 6                                                             UPCN468
 IN WELL AND PROFILES BENEATH WELL',                                       UPCN469
     1' (',3A2,')',//,11X,'T     C(WELL)',14X,'ELEVATION FOR C/(E)  (',   UPCN470
     2A2,') ',/,9X,'(',A2,')    E(WELL)')                                 UPCN471
      WRITE(NO,430) (C(K),K=LIM1,LIM2),(E(K),K=LIM1,LIM2)                  UPCN472
  430 FORMAT(24X,6(F8.1,1X),/,24X,6('     (',F3.1,')'))                    UPCN473
  431 DO 455  I=1,IT                                                       UPCN474
       DO 449  K=LIM1,LIM2                                                 UPCN475
          KFLG(K) =  KHAR1                                                 UPCN476
          IF(Z(I,K).GT.ZCR) KFLG(K) = KHAR2                                UPCN477
  449  CONTINUE                                                            UPCN478
       WRITE(NO,450) T(I),CW(I),(Z(I,K),KFLG(K),K=LIM1,LIM2),EW(I)         UPCN479
  450  FORMAT(/,6X,F8.2,F9.2,1X,6(F8.1,A1),/,16X,('(',F6.4,')'))           UPCN480
  455 CONTINUE                                                             UPCN481
      WRITE(NO,457) ZCR,IL                                                 UPCN482
  457 FORMAT('0',2X,'* NOTE: THE DISPERSION CONCEPT SHOULD BE LIMITED',    UPCN483
     1' TO THE ZONE BELOW',/,11X,' THE CRITICAL ELEVATION OF',F12.4,A3)    UPCN484
      IF(LIM2.EQ.11) GO TO 700                                             UPCN485
      LIM1 = 6                                                             UPCN486
      LIM2 = 11                                                            UPCN487
      GO TO 447                                                            UPCN488
C                                                                          UPCN489
C                                                                          UPCN490
```

```
NCENTRATION IN PUMPED WATER                                                    UPCN491
C                                                                              UPCN492
  500 CONTINUE                                                                 UPCN493
      IEDIT = 4                                                                UPCN494
      IF(KCON.NE.2) GO TO 176                                                  UPCN495
  510 WRITE(NO,515) IM1,IM2,IM3                                                UPCN496
  515 FORMAT('O',2X,'ENTER MAXIMUM PERMISSIBLE CONCENTRATION IN PUMPED',       UPCN497
     1' WATER (',3A2,')',/,3X,'?')                                             UPCN498
  516 READ(NI,50) CMAX                                                         UPCN499
      IF(CMAX.GE.CB.AND.CMAX.LT.CO) GO TO 518                                  UPCN500
      WRITE(NO,517)                                                            UPCN501
  517 FORMAT(3X,'CONCENTRATION MUST BE GREATER THAN OR EQUAL TO CB',/,         UPCN502
     13X,'AND LESS THAN CO -- REENTER',                                        UPCN503
     2/,3X,'?')                                                                UPCN504
      GO TO 516                                                                UPCN505
  518 CONTINUE                                                                 UPCN506
C                                                                              UPCN507
C                                                                              UPCN508
      SO = SO2/2.0                                                             UPCN509
      EMAX = (CMAX-CB)/(CO-CB)                                                 UPCN510
      EXCR = EMAX/(0.5*PHI)                                                    UPCN511
      ELIM = 0.0                                                               UPCN512
      IF(SO.GT.0.0) ELIM = 0.5*ERFC(XCR/(1.1414214*SO))                        UPCN513
      CINIT = 0.5*PHI*ELIM*(CO-CB) + CB                                        UPCN514
      IF(EXCR.LT.ELIM) GO TO 550                                               UPCN515
      IF(EXCR.LE.0.0.OR.EXCR.GE.1.0) GO TO 520                                 UPCN516
      CERF = 2.0*EXCR                                                          UPCN517
      XLIM1 = 3.0                                                              UPCN518
      XLIM2 = 0.0                                                              UPCN519
      CALL IERFC(CERF,ARG,XLIM1,XLIM2)                                         UPCN520
      B = -4.0*ARG*ARG*DM - 2.0*XCR                                            UPCN521
      CON = -2.0*ARG*ARG*SO*SO + XCR*XCR                                       UPCN522
      GO TO 521                                                                UPCN523
  520 B = -12.5*DM - 2.0*XCR                                                   UPCN524
      CON = -6.25*SO*SO + XCR*XCR                                              UPCN525
  521 CONTINUE                                                                 UPCN526
      ROOT = B*B - 4.0*CON                                                     UPCN527
      IF(ROOT.LT.0.0) GO TO 650                                                UPCN528
      XBAR1 = (-B-(ROOT**0.5))/2.0                                             UPCN529
      XBAR2 = (-B+(ROOT**0.5))/2.0                                             UPCN530
      XBAR = XBAR1                                                             UPCN531
      IF(EXCR.GT.0.5) XBAR=XBAR2                                               UPCN532
      ZMAX = XBAR + ZO                                                         UPCN533
      JFLG = KHAR1                                                             UPCN534
      IF(ZMAX.GT.ZCR) JFLG=KHAR2                                               UPCN535
      QMAX = 6.283185*((RHOS-RHOF)/RHOF)*KX*D*XBAR                             UPCN536
C                                                                              UPCN537
      WRITE(NO,280) (A(I),I=1,30),RHOF,RHOS,PO,IL,ITU,KX,IL,ITU,KZ             UPCN538
      WRITE(NO,281) IL,ZO,IL,D,THETA,IL,XCR,IL,ZCR                             UPCN539
      WRITE(NO,290) IM1,IM2,IM3,CO,IM1,IM2,IM3,CB,IL,SO2,IL,DM,PHI             UPCN540
      WRITE(NO,525) IM1,IM2,IM3,CMAX,EMAX,IL,ZMAX,JFLG,IL,ITU,QMAX             UPCN541
  525 FORMAT('O',5X,'MAXIMUM CONCENTRATION IN PUMPED WATER (',3A2,')',         UPCN542
     11X,F10.4,/,                                                              UPCN543
     26X,'MAXIMUM RELATIVE CONCENTRATION',17X,F10.4,/,                         UPCN544
     36X,'MAXIMUM INTERFACE ELEVATION (',A2,')',15X,F10.4,A1,/,                UPCN545
     46X,'MAXIMUM PERMISSIBLE PUMPING RATE (CU ',A2,'/',A2,')',4X,             UPCN546
     5F10.4)                                                                   UPCN547
      IF(ZMAX.LE.ZCR) GO TO 530                                                UPCN548
      WRITE(NO,457) ZCR,IL                                                     UPCN549
      CLIM = 0.5*(0.5*PHI)*(CO-CB) + CB                                        UPCN550
      WRITE(NO,527) CLIM,IM1,IM2,IM3                                           UPCN551
  527 FORMAT(3X,'(MAXIMUM CONCENTRATIONS IN PUMPED WATER LESS THAN',           UPCN552
     1F10.2,A3,2A2,') ',/,'O')                                                 UPCN553
C                                                                              UPCN554
  530 WRITE(NO,535) IL,ITU                                                     UPCN555
  535 FORMAT('O',2X,'ENTER OPTIONAL PUMPING RATE (CU ',A2,'/',A2,')',          UPCN556
     1/,3X,'?')                                                                UPCN557
      READ(NI,50) QP                                                           UPCN558
      IF(QP.LE.QMAX) GO TO 600                                                 UPCN559
      TIME = ((2.*PO*D)/(((RHOS-RHOF)/RHOF)*KZ))*((1./(1.-QMAX/QP))-1.)        UPCN560
```

```
      WRITE(NO,545) IL,ITU,QP,ITU,TIME                                  UPCN561
  545 FORMAT('O',2X,'PUMPING RATE (CU ',A2,'/',A2,')',6X,F10.4,/        UPCN562
     1                                                                  UPCN563
      GO TO 530                                                         UPCN564
C                                                                       UPCN565
  550 WRITE(NO,555) SO2,IL,CINIT,IM1,IM2,IM3                            UPCN566
  555 FORMAT('O',2X,'CMAX EXCEEDED FOR AN INITIAL TRANSITION ',         UPCN567
     1'ZONE WIDTH OF',F10.4,A3,/,                                       UPCN568
     23X,'INITIAL CONCENTRATION IN PUMPED WATER IS',F10.4,A3,2A2)       UPCN569
  600 CONTINUE                                                          UPCN570
      GO TO 700                                                         UPCN571
C                                                                       UPCN572
  650 WRITE(NO,655)                                                     UPCN573
  655 FORMAT(3X,'IMAGINARY ROOT OBTAINED FOR XMAX')                     UPCN574
      GO TO 700                                                         UPCN575
C                                                                       UPCN576
C                                                                       UPCN577
C ***** SECTION III -- PROBLEM REDEFINITION AND CONTROL OF EXECUTION    UPCN578
C                                                                       UPCN579
C                                                                       UPCN580
  700 CONTINUE                                                          UPCN581
      IEDIT = 2                                                         UPCN582
      XCR = THETA*D                                                     UPCN583
      ZCR = XCR + ZO                                                    UPCN584
      QMAXSS = 6.283185*((RHOS-RHOF)/RHOF)*KX*D*XCR                     UPCN585
      WRITE(NO,705)                                                     UPCN586
  705 FORMAT(//,3X,'ENTER NEXT COMMAND',/,3X,'?')                       UPCN587
  710 READ(NI,715) NEXT                                                 UPCN588
  715 FORMAT(A2)                                                        UPCN589
C                                                                       UPCN590
      DO 720  I=1,20                                                    UPCN591
       IF(NEXT.EQ.IC(I)) GO TO 730                                      UPCN592
  720 CONTINUE                                                          UPCN593
      WRITE(NO,725)                                                     UPCN594
  725 FORMAT(3X,'ERROR IN LAST COMMAND -- REENTER',/,3X,'?')            UPCN595
      GO TO 710                                                         UPCN596
  730 GO TO (29,70,90,111,120,129,145,188,215,235,255,                  UPCN597
     1311,312,322,302,275,300,500,1,1000),I                             UPCN598
C                                                                       UPCN599
 1000 STOP                                                              UPCN600
      END                                                               UPCN601
```

APPENDIX D

Listing of Function Subroutines

```
      FUNCTION ERFC(Z)                                                  ERFC001
C     RATIONAL APPROXIMATION OF THE COMPLIMENTARY ERROR FUNCTION        ERFC002
C     SEE SECTION 7.1 OF ABRAMOWITZ AND STEGUN (1966)                   ERFC003
      IF(ABS(Z).GT.7.5) GO TO 30                                        ERFC004
C     THE FOLLOWING IDENTITIES ARE USED TO HANDLE NEGATIVE ARGUMENTS    ERFC005
C        ERFC( Z) = 1 - ERF(Z)                                          ERFC006
C        ERFC(-Z) = -ERFC(Z) = 1 + ERF(Z)                               ERFC007
C                                                                       ERFC008
      X = Z                                                             ERFC009
C     NEGATIVE ARGUMENTS                                                ERFC010
      IF (Z.LT.0.0) X = -Z                                              ERFC011
      ERFC =      1.0/((1.0 + 0.070523*X + 0.042282*(X**2)              ERFC012
     1                  + 0.009270*(X**3) + 0.000152*(X**4)             ERFC013
     2                  + 0.000276*(X**5) + 0.000043*(X**6))**16)       ERFC014
C     NOTE: 2-ERF(X) = ERFC(-X) = ERFC(Z) FOR Z<0                       ERFC015
      IF (Z.LT.0.0) ERFC = 2.0 - ERFC                                   ERFC016
      RETURN                                                            ERFC017
C                                                                       ERFC018
C     FOR Z>7.5, ERFC(Z)<2.32E-22 AND IS SET TO 0                       ERFC019
   30 ERFC = 0.0                                                        ERFC020
C     FOR Z < -7.5  ERFC(Z) IS SET TO 2.0                               ERFC021
      ERFC = 2.0                                                        ERFC022
      RETURN                                                            ERFC023
      END                                                               ERFC024
```

```
C     INVERSE COMPLIMENTARY ERROR FUNCTION                          IERF001
      SUBROUTINE IERFC(CERF,X,XLH,XRH)                              IERF002
C     INVERSE COMPLIMENTARY ERROR FUNCTION                          IERF003
C     A REGULA FALSI ROOT-FINDING TECHNIQUE IS USED TO              IERF004
C     LOCATE X FOR A SPECIFIED VALUE OF CERF.  XLH AND              IERF005
C     XRH DEFINE THE INITIAL SEARCH INTERVAL.                       IERF006
C     SEE CARNAHAN, LUTHER AND WILKES (1969) FOR A                  IERF007
C     DISCUSSION OF THE METHOD.                                     IERF008
      FLH = CERF - ERFC(XLH)                                        IERF009
      FRH = CERF - ERFC(XRH)                                        IERF010
   10 XEST = (XLH*FRH - XRH*FLH)/(FRH - FLH)                        IERF011
      FEST = CERF - ERFC(XEST)                                      IERF012
      IF(ABS(FEST).GT.1.0E-04) GO TO 20                             IERF013
      X = XEST                                                      IERF014
      RETURN                                                        IERF015
   20 IF(FEST.GT.0.0.AND.FRH.GT.0.0) GO TO 30                       IERF016
      XLH = XEST                                                    IERF017
      FLH = CERF - ERFC(XLH)                                        IERF018
      GO TO 10                                                      IERF019
   30 XRH = XEST                                                    IERF020
      FRH = CERF - ERFC(XRH)                                        IERF021
      GO TO 10                                                      IERF022
      END                                                           IERF023
```